
ENGINEERING PROBABILITY

HOMEWORK # 10:
Posted on 04/04/2018

Please work out the **ten** (10) problems stated below – BT refers to the text: D.P. Bertsekas and J.N. Tsitsiklis, Introduction to Probability (Second Edition), Athena Scientific (2008). Problem **1.55** (BT) refers to Problem 55 for Chapter 1 of BT (to be found at the end of Chapter 1). **Show** work and **explain** reasoning.

1.

If the rv $X : \Omega \to \mathbb{R}$ is a standard Gaussian rv, i.e., $X \sim \mathrm{N}(0,1)$, show that the rv $Y : \Omega \to \mathbb{R}$ given by

$$Y \equiv \begin{cases} 0 & \text{if } X \leq 0 \\ \\ 1 & \text{if } X > 0 \end{cases}$$

is a discrete rv and compute its pmf.

2.

Problem **3.2** (BT)

3.

Problem **3.5** (BT)

4.

Problem **3.7** (BT)

5.

Problem **3.11** (BT)

6.

This problem should alert you to the centrality played by the standard normal distribution (i.e., $\mu = 0$ and $\sigma^2 = 1$): Problem **3.12** (BT)

7.

Let S be a bounded region of $\mathbb{R}^2$ with $\text{Area}(S) > 0$. The pair of rvs X and Y are said to be uniformly distributed on S if

$$\mathbb{P}\left[(X,Y) \in B\right] = \int_B f(x,y)dxdy, \quad B \subseteq \mathbb{R}^2$$

where $f : \mathbb{R} \to \mathbb{R}_+$ is given by

$$f(x,y) = \begin{cases} \frac{1}{\text{Area}(S)} & \text{if } (x,y) \in S \\ \\ 0 & \text{if } (x,y) \notin S \end{cases}$$

Show that the rvs X and Y are both (absolutely) continuous rvs, and identify their probability density functions $f_X, f_Y : \mathbb{R} \to \mathbb{R}_+$ in the following cases:

7.a. S is the unit square $[0,1] \times [0,1]$

7.b. S is the diamond

$$\{(x,y) \in [-1,1] \times [-1,1] : \ |y| \leq |x|\}.$$

7.c. S is the unit disk

$$\left\{(x,y) \in \mathbb{R}^2 : \ x^2 + y^2 \leq 1\right\}.$$

8. ______________________________

Problem **3.15** (BT)

9. ______________________________

The rv $X : \Omega \to \mathbb{R}$ is of continuous type with probability density function $f_X : \mathbb{R} \to \mathbb{R}_+$ given by

$$f_X(x) = \begin{cases} 0 & \text{if } x < 1 \\ \\ Cx^{-4} & \text{if } 1 \leq x \end{cases}$$

for some constant $C > 0$.

9.a. What should be the value of C?

9.b. Find $\mathbb{P}[0.5 < X < 2]$ and $\mathbb{P}[2 < X < 4]$

9.c. Find the CDF $F_X : \mathbb{R} \to [01,]$ of the rv X.

9.d. Find $\mathbb{E}[X^n]$ for *all* $n = 1, 2, \ldots$

10. ______________________________

The rv $X : \Omega \to \mathbb{R}$ is of continuous type with probability density function $f_X : \mathbb{R} \to \mathbb{R}_+$ given by

$$f_X(x) = \begin{cases} 0 & \text{if } x < 1 \\ \\ \frac{1}{2}x^{-\frac{3}{2}} & \text{if } 1 \leq x. \end{cases}$$

10.a. Find $\mathbb{P}[X > 10]$

10.b. Find the CDF $F_X : \mathbb{R} \to [01,]$ of the rv X.

10.c. Find $\mathbb{E}[X]$.

10.d. Find $\mathbb{E}\left[X^{\frac{1}{4}}\right]$.
