
ENGINEERING PROBABILITY

HOMEWORK # 13:
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Please work out the **ten** (10) problems stated below – BT refers to the text: D.P. Bertsekas and J.N. Tsitsiklis, Introduction to Probability (Second Edition), Athena Scientific (2008). Problem **1.55** (BT) refers to Problem 55 for Chapter 1 of BT (to be found at the end of Chapter 1). **Show** work and **explain** reasoning.

1.

A very important problem[1] – Let X and Y be independent rvs with X (resp. Y) normally distributed[2] with mean a (resp. b) and variance $\alpha^2 > 0$ (resp. $\beta^2 > 0$). Show by a completion of squares argument that the rv $Z = X + Y$ is also normally distributed, and determine its mean and variance.

Consider now the following situation: The n rvs $X_1, \ldots, X_n$ are mutually independent rvs. For each $i = 1, \ldots, n$, the rv X_i is normally distributed with mean a_i and variance $\alpha_i^2 > 0$. Show that the rv $X_1 + \ldots + X_n$ is also normally distributed, and identify its parameters.

2.

Let X, Y and Z be independent and identically distributed rvs of the discrete type with

$$\mathbb{P}[X = 1] = \mathbb{P}[X = 2] = \frac{1}{2}.$$

2.a Find the pmf of the rv $A = XYZ$.

2.b Find the pmf of the rv $B = XY + YZ + ZX$.

3.

Problems **4.17**, **4.18** and **4.19** (BT)

4.

Let X and Y be two independent, each normally distributed with zero mean and unit variance.

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[1] See also Problem **4.28** (BT)

[2] A rv is normally distributed is just another way to say that it is a gaussian or normal rv.

4.a. Find the joint distribution of the rvs U and V where

$$U = X, \ V = \begin{cases} \frac{Y}{X} & \text{if } X \neq 0 \\ \\ 0 & \text{if } X = 0. \end{cases}$$

4.b. Determine whether this joint distribution is of continuous type. In the affirmative, identify the joint pdf.

4.c. Are the rvs U and V independent? Explain!

5.

If X and Y are independent rvs which each uniformly distributed on $(0, 1)$, compute a closed form expression for the moment

$$\mathbb{E}\left[\frac{\max(X, Y)}{\min(X, Y)} \mathbf{1}\left[\min(X, Y) \neq 0\right]\right].$$

6.

Consider two rvs X and Y which are independent and exponentially distributed, with parameter $\lambda > 0$ and $\mu > 0$, respectively. The rv T is the function of X and Y given by

$$T = \begin{cases} \frac{Y}{X} & \text{if } X \neq 0 \\ \\ 0 & \text{if } X = 0. \end{cases}$$

6.a. Compute the cumulative distribution function $F_T : \mathbb{R} \to [0, 1]$ of T, namely

$$F_T(t) = \mathbb{P}\left[T \leq t\right], \quad t \in \mathbb{R}$$

6.b. Show that the rv T is of continuous type and identify its probability density function $f_T : \mathbb{R} \to \mathbb{R}_+$

6.c. Compute the first moment $\mathbb{E}\left[T\right]$. Carefully explain your answer!

7.

With positive integer $n \geq 3$, let $R_1, R_2, \ldots, R_n$ be n mutually independent and identically distributed rvs, each exponentially distributed with parameter $\lambda > 0$.

7.a. Explicitly compute the probability $\mathbb{P}\left[R_1 > 2R_2\right]$.

7.b. Explicitly compute the probability $\mathbb{P}\left[R_1 > 2R_2 > 3R_3\right]$.

7.c. Does the probability $\mathbb{P}\left[R_1 > 2R_2 > \ldots > (n-1)R_{n-1} > nR_n\right]$ depend on the parameter λ? Explain your answer!

7.d. Compute the probability $\mathbb{P}\left[R_1 > R_2 > \ldots > R_{n-1} > R_n\right]$.

8.

Problem **4.22** (BT).

9.

Problem **4.23** (BT).

10.

Problem **4.24** (BT).