# ENEE 324-01 / SPRING 2018

## Homework Solutions # 4

(1) Problem 1

Check the sol^n manual

Problem 2

Problem 3 Done in recitation 4.

Problem 4

" 5

Check the sol^n manual.

" 7

" 8

" 9

" 10

## Problem 6

(6.a.) $\Omega = \left\{ \text{All possible combinations of } K \text{ distinct tickets out of the } N \text{ distinct tickets} \right\}$

$$|\Omega| = \binom{N}{K} = \frac{N!}{(N-K)!\,K!}$$

Since $|\Omega| < \infty$, $\mathcal{F}$ can be taken as $\mathcal{P}(\Omega)$.

By the uniformity assumption

$$\mathbb{P}[\{\text{singleton}\}] = \frac{1}{|\Omega|} = \frac{(N-K)!\,K!}{N!}$$

(b) If the event of Mr. Noone buying at-least one winning ticket is $A$,

$$\mathbb{P}(A^c) = \binom{N-W}{K} \times \frac{1}{|\Omega|}$$

(the probability of Mr. Noone buying no winning tickets)

$$= \frac{(N-W)!}{K!\,(N-W-K)!} \times \frac{(N-K)!\,K!}{N!}$$

$$= \frac{(N-W)!\,(N-K)!}{(N-W-K)!\,N!}$$

$$\therefore \quad \mathbb{P}(A) = 1 - \mathbb{P}(A^c)$$

$$= 1 - \frac{(N-W)!\,(N-K)!}{(N-W-K)!\,N!} \quad //$$

(c) $$\mathbb{P}(A) = 1 - \frac{(N-W)(N-W-1)\cdots 2\times 1 \times (N-K)(N-K-1)\cdots \times 2 \times 1}{(N-W-K)(N-W-K-1)\cdots 2\times 1 \times N \times (N-1) \times \cdots\cdots 2 \times 1}$$

$$= 1 - \frac{(N-W)(N-W-1)\times \cdots \times (N-W-K+1)}{N \times (N-1) \times \cdots\cdots \times (N-K+1)}$$

(Cancelling out common terms)

$$\mathbb{P}(A) = 1 - \frac{\left(1 - W/N\right)\left(1 - \frac{W+1}{N}\right) \cdots \left(1 - \frac{W+K-1}{N}\right)}{(1)\left(1 - 1/N\right) \times \cdots \times \left(1 - \frac{K-1}{N}\right)}$$

(Dividing numerator and denomenator by $N^K$.)

Now, take the limit $N \to \infty$ for fixed $K$ and $W$

$$\lim_{N \uparrow \infty} \mathbb{P}(A) = 1 - \lim_{n \to \infty} \frac{\left(1 - W/N\right)\left(1 - \frac{W+1}{N}\right) \cdots \left(1 - \frac{W+K-1}{N}\right)}{\left(1 - 1/N\right)\left(1 - 2/N\right) \cdots \left(1 - \frac{K-1}{N}\right)}$$

$$= 1 - \frac{1 \times 1 \cdots \times 1}{1 \times 1 \cdots \times 1}$$

$$= 1 - 1$$

$$= 0 \; //$$

Hence when $K$ and $W$ are fixed, as $N$ becomes larger, the probability of purchasing at-least one winning ticket becomes smaller. This is intuitive as well, because when $W$ is fixed, with $N$ increasing, the proportion of winning tickets reduces.