
ENGINEERING PROBABILITY

HOMEWORK # 5:
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Please work out the **ten** (10) problems stated below – BT refers to the text: D.P. Bertsekas and J.N. Tsitsiklis, Introduction to Probability (Second Edition), Athena Scientific (2008). Problem **1.55** (BT) refers to Problem 55 for Chapter 1 of BT (to be found at the end of Chapter 1). **Show** work and **explain** reasoning.

1.

With $(\Omega, \mathcal{A}, \mathbb{P})$ a probability model, let $A_1, \ldots, A_n$ be a collection of mutually independent events. If I is *any* subset of $\{1, \ldots, n\}$ with $|I| \geq 2$, show that the events $\{A_i,\ i \in I\}$ are also mutually independent.

2.

With $(\Omega, \mathcal{A}, \mathbb{P})$ a probability model, we are told that the events A, B, C and D are mutually independent. Determine whether the following events are mutually independent:

2.a. $A \cup B^c$ and C

2.b. $A \cap B$ and $C \cup D$

2.c. $A \cup B$ and B

2.d. $(A \cup B) \cap D$ and C

2.e. $(A \cap B) \cup D$ and C

3.

Problem **1.36** (BT)

4.

Problem **1.38** (BT)

5.

Problem **1.39** (BT)

6.

Problem **1.40** (BT)

7.

Consider n distinct random experiments $\mathcal{E}_1, \ldots, \mathcal{E}_n$, where for each $i = 1, \ldots, n$, experiment $\mathcal{E}_i$ is modeled by the probability model $(\Omega_i, \mathcal{F}_i, \mathbb{P}_i)$.

You are told that these experiments are "mutually independent." The "joint" or combined experiment

$$\mathcal{E} = \mathcal{E}_1 \times \ldots \times \mathcal{E}_n?$$

is usually modeled by the triple $(\Omega, \mathcal{F}, \mathbb{P})$ where the sample space Ω is given by

$$\Omega = \Omega_1 \times \ldots \times \Omega_n$$

and the probability assigment $\mathbb{P}$ should have the following form: For events $A_1, \ldots, A_n$ (associated with $\mathcal{E}$) which have the "product form"

$$A_i = \Omega_1 \times \ldots \times \Omega_{i-1} \times B_i \times \Omega_{i+1} \times \ldots \times \Omega_n, \quad i = 1, 2, \ldots, n$$

where B_i is *any* event for $\mathcal{E}_i$, we *stipulate* that

$$\mathbb{P}\left[A_1 \cap \ldots \cap A_n\right] = \prod_{i=1}^{n} \mathbb{P}_i[B_i] \tag{1.1}$$

7.a. With this notation show that

$$A_1 \cap \ldots \cap A_n = \times_{i=1}^{n} B_i, \qquad \begin{matrix} B_i \in \mathcal{F}_i \\ i = 1, 2, \ldots, n \end{matrix}$$

7.b. Show that with this definition, "product form" events as above are always *mutually* independent under $\mathbb{P}$.

8.

You are told about n distinct random experiments $\mathcal{E}_1, \ldots, \mathcal{E}_n$ whose sample spaces $\Omega_1, \ldots, \Omega_n$ are finite.

You are invited to consider the joint experiment $\mathcal{E} = \mathcal{E}_1 \times \ldots \times \mathcal{E}_n$ and decide that it is appropriate to take the probability assignment $\mathbb{P}$ to be the *uniform* probability assigment on the sample space $\Omega = \Omega_1 \times \ldots \times \Omega_n$.

Use this information to determine (i) the probability model for each of the experiments $\mathcal{E}_1, \ldots, \mathcal{E}_n$, and (ii) whether the model you have decided to use is compatible with the n experiemnts being mutually independent (in the sense of Problem **7**).

9.

You have a beautiful coin in your hand. Each time you flip the coin, Head (resp. Tail) appears with probability p (resp. $1 - p$) (where $0 < p < 1$). You are told to flip this coin infinitely times, under *identical* and *independent* conditions.

9.a. This defines a random experiment $\mathcal{E}$. Describe a probability model for it under the conditions mentionned above!

9.b. Under this model, what would be the probability that you never get Tail?

9.c. Under this model, compute the probability of getting Tail before ever getting Head.

10.

Let $\mathcal{X}$, $\mathcal{Y}$ and $\mathcal{Z}$ be three arbitrary sets. We are given two functions $f : \mathcal{X} \to \mathcal{Y}$ and $g : \mathcal{Y} \to \mathcal{Z}$. Define the *composition* function $h : \mathcal{X} \to \mathcal{Z}$ by

$$h(x) = g(f(x)), \quad x \in \mathcal{X}.$$

Show that

$$h^{-1}(F) = f^{-1}\left(g^{-1}(F)\right), \quad F \subseteq \mathcal{Z}.$$