
ENGINEERING PROBABILITY

HOMEWORK # 7:
Posted on 03/07/2018

Please work out the **ten** (10) problems stated below – BT refers to the text: D.P. Bertsekas and J.N. Tsitsiklis, Introduction to Probability (Second Edition), Athena Scientific (2008). Problem **1.55** (BT) refers to Problem 55 for Chapter 1 of BT (to be found at the end of Chapter 1). **Show** work and **explain** reasoning.

1.

Problem **2.16** (BT)

2.

Problem **2.18** (BT)

3.

Problem **2.20** (BT)

4.

Problem **2.22** (BT)

5.

Problem **2.24** (BT)

6.

Problem **2.26** (BT)

7.

The discrete rvs $X : \Omega \to \mathbb{R}$ and $Y : \Omega \to \mathbb{R}$ are defined on some probability triple $(\Omega, \mathcal{F}, \mathbb{P})$ with support $S_X = \{0, 1 \ldots, n\}$ (for some positive integere n) and $S_Y = \mathbb{N}$. It is known that

$$\mathbb{P}[X = x, Y = y] = C \binom{n}{x} (1-p)^x p^{n-x} p^y, \qquad \begin{matrix} x = 0, 1, \ldots, n \\ y = 0, 1, \ldots \end{matrix}$$

with $0 < p < 1$ for some $C > 0$.

7.a. How should $C > 0$ be selected? Explain and find the appropriate value(s).

7.b. Find the pmf of the rv X and the pmf of the rv Y.

7.c. Is the rv $\Omega \rightarrow: \omega \rightarrow (X(\omega), Y(\omega))$ a discrete rv? Explain! If yes, identify its support.

8.

The following facts concerning power series are well known: With

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}, \quad |x| < 1,$$

differentiation and summation can be interchanged. Specifically, for each $n = 1, 2, \ldots$, we have

$$\begin{aligned} \frac{d^n}{dx}\left(\sum_{k=0}^{\infty} x^k\right) &= \sum_{k=0}^{\infty} k(k-1)\ldots(k-n+1)x^{k-n} \quad \text{(Term-by-term differentiation)} \\ &= \sum_{k=n}^{\infty} k(k-1)\ldots(k-n+1)x^{k-n} \quad |x| < 1, \end{aligned}$$

Use this fact to compute $\mathbb{E}[X]$, $\mathbb{E}[X^2]$ and $\text{var}[X]$ when X is the geometric rv with pmf

$$\mathbb{P}[X = k] = (1-p)^{k-1}p, \quad k = 1, 2, \ldots$$

9.

Consider N discrete rvs $X_i : \Omega \rightarrow \mathbb{R}$, $i = 1. \ldots, N$, defined on the probability triple $(\Omega, \mathcal{F}, \mathbb{P})$, with

$$\mathbb{P}[X_i \in S_i] = 1, \quad i = 1, \ldots, N$$

where for each $i = 1, 2, \ldots, N$, the support S_i of the rv X_i is a countable subset of $\mathbb{R}$.

9.a. Define the rv $Z : \Omega \rightarrow \mathbb{R}$ by

$$Z = \prod_{i=1}^{N} X_i$$

Prove that the rv Z is a discrete rv and identify its support S_Z.

9.b. How would you compute the pmf of the rv Z? Does the knowledge of the pmfs of the rvs $X_1, \ldots, X_N$ suffice? Explain!

10.

Consider N discrete rvs $X_i : \Omega \rightarrow \mathbb{R}$, $i = 1. \ldots, N$, with

$$\mathbb{P}[X_i \in S_i] = 1, \quad i = 1, \ldots, N$$

where S_i is a countable subset of $\mathbb{R}$.

10.a. Show that

$$\mathbb{P}[X_1 \in S_1, \ldots, S_N \in S_N] = 1$$

10.b. For any mapping $g : \mathbb{R}^N \to \mathbb{R}$, define the rv $Z : \Omega \to \mathbb{R}^N$ given by

$$Z = g(X_1, \ldots, X_N).$$

Show that the rv Z is a discrete rv, and determine its pmf in terms of the joint pmf of the rvs $X_1, \ldots, X_N$.

10.c. Show that

$$\mathbb{E}[Z] = \sum_{(x_1,\ldots,x_N)\in S_1\times\ldots\times S_N} g(x_1, \ldots, x_N) p_{X_1,\ldots,X_N}(x_1, \ldots, x_N)$$

provided

$$\sum_{(x_1,\ldots,x_N)\in S_1\times\ldots\times S_N} |g(x_1, \ldots, x_N)| p_{X_1,\ldots,X_N}(x_1, \ldots, x_N) < \infty.$$
