
ENGINEERING PROBABILITY

HOMEWORK # 8:
Posted on 03/21/2018

Please work out the **ten** (10) problems stated below – BT refers to the text: D.P. Bertsekas and J.N. Tsitsiklis, Introduction to Probability (Second Edition), Athena Scientific (2008). Problem **1.55** (BT) refers to Problem 55 for Chapter 1 of BT (to be found at the end of Chapter 1). **Show** work and **explain** reasoning.

1.

Problem **2.38** (BT)

2.

Problem **2.39** (BT)

3.

Let X be a Poisson rv with parameter $\lambda > 0$. You are told that the rv Y is a discrete $\mathbb{N}$-valued rv with pmf

$$\mathbb{P}[Y = y] = \mathbb{P}[X = y | X > 0], \quad y = 0, 1, \ldots$$

3.a. Give a *closed-form* expression for the expectation $\mathbb{E}[Y]$. Which of $\mathbb{E}[X]$ and $\mathbb{E}[Y]$ is larger? Is the conclusion surprising? Explain.

3.b. Give a *closed-form* expression for the variance $\text{Var}[Y]$ of Y.

3.c. Compute the ratio

$$\frac{\text{Var}[Y]}{\text{Var}[X]}$$

and decide whether $\text{Var}[X]$ is greater than $\text{Var}[Y]$.

4.

Consider two discrete $\mathbb{N}$-valued rvs X_1 and X_2 with pmfs given by

$$\mathbb{P}[X_k = x] = a_k(1 - a_k)^x, \qquad \begin{matrix} x = 0, 1, \ldots \\ k = 1, 2 \end{matrix}$$

where $0 < a_1, a_2 < 1$. The rvs X_1 and X_2 are assumed to be independent.

4.a. Compute the probabilities

$$\mathbb{P}\left[X_1 + X_2 = y\right], \quad y = 0, 1, \ldots$$

Explain your steps!

From now on assume that $a_1 = a_2 = a$.

4.b. Evaluate the conditional probabilities

$$\mathbb{P}\left[X_1 = x | X_1 + X_2 = y\right], \quad x = 0, \ldots, y.$$

Is this a well-known pmf, and if so, which one? Explain!

4.c. Compute the conditional expectations

$$\mathbb{E}\left[X_1 | X_1 + X_2 = y\right], \quad y = 0, 1, \ldots$$

5.

Problem **2.40** (BT)

6.

Problem **2.41** (BT)

7.

Let X and Y be two Poisson rvs with parameters $\lambda > 0$ and $\mu > 0$, respectively. Assume X and Y to be independent.

7.a. Compute

$$\mathbb{P}\left[X + Y = z | X = x\right], \quad x, y = 0, 1, \ldots$$

7.b. Compute

$$\mathbb{P}\left[X = x | X + Y = z\right], \quad x, y = 0, 1, \ldots$$

8.

For some positive integer n, let $\xi_1, \ldots, \xi_n$ denote n rvs which all take values in the set of integers $\{0, 1, \ldots, 9\}$, i.e., $\mathbb{P}\left[\xi_k \in \{0, 1, \ldots, 9\}\right] = 1$ for each $k = 1, \ldots, n$. The rv X is now defined in terms of these rvs through the decimal expansion

$$X := 0.\xi_1 \xi_2 \ldots \xi_n.$$

8.a. Compute the expected value $\mathbb{E}\left[X\right]$ of X in terms of the expected values $\mathbb{E}\left[\xi_1\right], \ldots, \mathbb{E}\left[\xi_n\right]$. Give a closed form expression for $\mathbb{E}\left[X\right]$ when $\mathbb{E}\left[\xi_1\right] = \ldots = \mathbb{E}\left[\xi_n\right] = m$.

8.b. Specialize your answer in Part **8.a** when the rvs $\xi_1, \ldots, \xi_n$ are uniformly distributed on $\{0, 1, \ldots, 9\}$, i.e.,

$$\mathbb{P}\left[\xi_k = x\right] = \frac{1}{10}, \quad \begin{array}{c} x = 0, \ldots, 9 \\ k = 1, \ldots, n. \end{array}$$

What happens when n becomes large?

9. ___

Problem **2.25** (BT)

10. ___

Problem **2.31** (BT)
