HOMEWORK SET 1 (due back in class, **Thursday February 1**)
From Grimmett-Stirzaker (textbook)
Section 1.2 Problems 1 and 3
Section 1.3 Problem 1 (and generalize to the case when P(A) = a and P(B) = b)
Problem 4
Section 1.8 Problem 5

Readings (a) Appendix III, especially remarks on interpretation of probability on page 572; (b) Pages 1-15 of chapter 1. (c) PSK lecture notes – up to and including Counting Lecture page 5.

Comment: The empty set is the **impossible event**, since in an experiment an (elementary) outcome cannot fall in the empty set. The term **null event** is used for any event to which we have assigned probability zero. The impossible event is a null event. Converse is not true in general, as there may be probability assignments leading to zero probability for some non-empty subsets of the sample space.

HOMEWORK SET 2 (due back in class **Thursday, February 8**)
From Grimmett-Stirzaker (textbook)
Section 1.4 - Problems 2, 5 and 7
Section 1.5 - Problems 2 and 4
Section 1.8 - Problem 35

Additional Problems

P1 (Application of conditional probability): In a city with one hundred taxis, 1 is blue and 99 are green. A witness observes a hit-and-run by a taxi at night and recalls that the taxi was blue, so the police arrest the blue taxi driver who was on duty that night. The driver proclaims his innocence and hires you to defend him in court. You hire a scientist to test the witness' ability to distinguish blue and green taxis under conditions similar to the night of accident. The data suggests that the witness sees blue cars as blue 99% of the time and green cars as blue 2% of the time. Compute the probability that the driver is innocent.

P2 (Conditional Probability and Likelihood): There are four dice in a drawer: one tetrahedron (4 sides), one hexahedron (i.e., cube, 6-sides), and two octahedra (8 sides). There are numbers written on each side of the dices (1,2,3,4 on the tetrahedron, 1,2,3,4,5,6 on the hexahedron and 1,2,3,4,5,6,7,8 on the octahedra). Your friend secretly grabs one of the four dice at random. Let S be the number of sides on the chosen die. Let R be the result of the roll.
(a) Use Bayes' rule to find $P(S = k \mid R = 3)$ for k = 4, 6, 8. Which die is most likely if R = 3?
(b) Which die is most likely if R = 6?
(c) Which die is most likely if R = 7? No computations are needed!

Readings - section 1.7 on worked examples from textbook

HOMEWORK SET 3 (due back in class on **Thursday, February 15**)
From Grimmett-Stirzaker (textbook)
Chapter 2
Section 2.1 Problems 1, 4, 5
Section 2.3 Problem 2, 3
Section 2.7 Problem 5, 6

Readings sections 2.1, 2.3, 2.4

HOMEWORK SET 4 (due back in class **Thursday, February 22)**

Section 2.3 - Problem 5
Section 2.4 - Problem 1(a) and 1(e)
Section 2.7 - Problem 4(a) and 4(d)
Section 3.1 - Problem 2

HOMEWORK SET 5 (due back in class **Thursday, March 8)**

Section 3.1 Problem 3
Section 3.2 Problems 1, 2, and 3
Section 3.3 Problem 2, 4
Section 3.11 Problem 7, 8, 10

READINGS - sections 3.1 to 3.6

HOMEWORK SET 6 (due in class on **Thursday**, **March 15**)

Section 3.4 Problems 1, 3, 5
Section 3.5 Problems 1, 2
Section 3.6 Problems 1, 2, 4, 5
Section 3.11 Problems 12, 13, 14, 16

READINGS – sections 3.6 – 3.8

HOMEWORK SET 7 (due in class on **Tuesday**, **April 3**)

Section 3.7 Problems 1, 5, 9
Section 3.8 Problems 1, 16 (16 should have been 6; 2018-04-02)
Section 3.11 Problems 6, 8 (remove 8; 2018-04-02)
Section 4.1 Problem 2
Section 4.14 Problems 1(a), 1(d) (limit as x tends to infinity), 4(a)

READINGS – section 4.1

HOMEWORK SET 8 (due in class on **Tuesday**, **April 17**)

Section 4.1 Problem 3 (try to find a suitable probability space for the random variable you describe)
Section 4.2 Problems 1, 3
Section 4.3 Problems 1, 3; understand what a median is and do 4
Section 4.4 Problems 3, 5
Section 4.5 Problems 4, 8
Section 4.6 Problems 2, 9
READINGS – section 4.1 through 4.7

HOMEWORK SET 9 (due in class on **Tuesday**, **April 24**)

Section 4.6 Problems 4, 6
Section 4.14 Problem 13
Section 5.1 Problems 2, 6
READINGS – section 4.7, 5.1, 5.7, PSK Lecture Notes, Lecture 10 (handwritten) on WLLN (see also Section 2.2 of book), Central Limit Theorem

HOMEWORK SET 10 (due in class on **Tuesday**, **May 1**)

Use standard Gaussian table distributed to class by email, **where (and if) necessary in problems 1, 2, and 3.**

Problem 1
A student uses a brand of pens whose lifetime of use is an exponential random variable with mean = 1 week. We are interested determining the minimum number of pens the student should buy at the beginning of a 15 week semester so that there is a probability of 0.99 that the student does not run out of working pens during the semester.

(a) Using the presentation on Tuesday April 17 (by Dipankar Maity in class, using Lecture Notes 10), can you justify applying the **Central Limit Theorem** to answer this question? Give details.
(b) If the answer to part (a) is **yes** (with justification spelled out), carry out the calculation to determine if **28** pens suffice. Use the standard Gaussian table for this purpose.
(c) If the student can only afford **25** pens, by how much does the probability of pen availability during the semester go down?

Problem 2
Let X and Y be standard Gaussian random variables with correlation 0.1. Consider the random variable Z = 3X + 4Y + 5.
What is the probability that Z lies between 0 and 10? If the correlation is given to be a general value **r** between -1 and + 1, can you suggest a formula for numerically calculating the same probability, incorporating the Gaussian table as well?

Problem 3
At a semiconductor manufacturing plant, in order to estimate the

probability P(A) of a chip-packaging failure event A, a succession of **n** Bernoulli trials is carried out and the associated relative frequency f(A) of A is observed.
(a) How large should the number **n** of trials be, in order to ensure a 95% chance that the observed relative frequency is within 0.1 of the underlying “true” probability p = P(A)?
(ASIDE: such true probabilities may be agreed upon in a contract, may depend in very complex ways on product design and manufacturing details and hard to predict – so estimates based on testing become necessary; test results and estimates may be subject to legal scrutiny by contracting parties)
(b) State clearly any theoretical results or principles you used to arrive at the answer in part (a). **HINT** - the number **n** is more than **300**.

Problem 4
In a certain town, matters of disagreement are occasionally settled by duels (if one actually takes place), or by a default outcome decided upon by a town council. In each such matter, each of the 2 opponents arrives at a random moment between 5:00 A.M. and 6:00 A.M. on an appointed decision day and leaves exactly 5 minutes later, honor served, unless both arrive within the 5 minute time interval in which case the matter in question is settled by a duel. What fraction of the matters of disagreement is settled peacefully by default decision of the town council?

Problem 5
If a stick is broken in two at random, what is the average length of the smaller piece? What is the average ratio of the smaller length to the larger?

Problem 6 Read Section 5.10 of the Textbook from page 193 till half of page 195 (before Local Central Limit Theorem), and do **problem 2** in that section.

HOMEWORK SET 11 (due in class **Tuesday, May 8**)

Problem 1

Let X_1 and X_2 be independent, non-negative, identically distributed continuous random variables, each with density

$$p_x(x) = 1000/x^2$$

for $x > 1000$.

What is the probability density of the ratio $Y = \dfrac{X_1}{X_2}$?

Clearly indicate the range R_Y of Y.

Problem 2

The random variables X and Y have joint probability density function given by

$$p_{X,Y}(x,y) = 2e^{-(x+y)} \text{ for } 0 \le y \le x < \infty, \quad \text{and} \quad p_{X,Y}(x,y) = 0, \text{ otherwise.}$$

Find the probability density function of $Z = X + Y$.

Problem 3

Let **X**(t) = **A**cos(**f**t) + **B**sin(**f**t) be a stochastic process where **f** is a constant, and **A** and **B** are independent Gaussian random variables each with mean **0** and standard deviation **s**. The time index set is the set of all real numbers.

(a) Find the mean and auto-covariance functions of the process **X**(t).
(b) Find joint pdf of X(t) and X(t+d).
(c) Is the process stationary? Weakly stationary?

From Section 4.7 of the Textbook
Problems 4, 5, 8
From Section 4.8 of the Textbook
Problems 1, 7