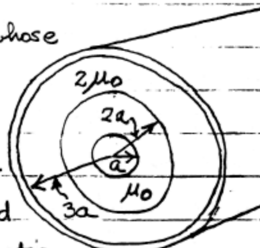
ENEE 380H FALL 2001 — FINAL EXAM (155 points total) — PROF. E. OTT

(SHOW ALL WORK LEADING TO YOUR ANSWERS.)

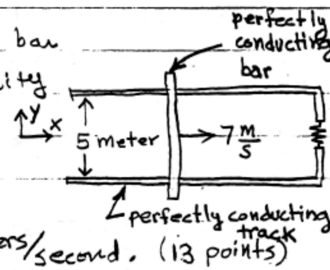
Problem 1: Consider a coaxial transmission line whose axis of symmetry coincides with the z-axis. The radius of the inner conductor is a; the inner radius of the outer conductor is $3a$; the region $a<r<2a$ is vacuum; and the region $2a<r<3a$ is occupied by an electrically nonconducting material of magnetic permeability $2\mu_0$.



(a) A current of 4 Amperes flows in the z-direction on the inner conductor and an equal and opposite current flows on the outer conductor. Find $\underline{B}$ and $\underline{H}$ in the region $a<r<2a$ and in the region $2a<r<3a$. (12 points)

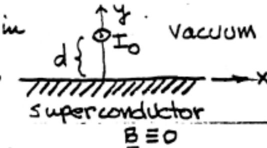
(b) Assuming that the current on the inner conductor is all on the surface $r=a$, and that the current on the outer conductor is all on the surface $r=3a$, find the inductance per unit length for the transmission line. (13 points)

Problem 2: (a) The sliding perfectly conducting bar in the figure is moving with a constant velocity of $7\underline{a}_x$ meters/second through a magnetic field of $\underline{B} = 3\underline{a}_z$ Webers/m^2. $R = 13\Omega$. Find the voltage across the resistor R. Find the force required to move the bar at 7 meters/second. (13 points)



(b) Using Maxwell's equations only, derive the equation expressing the conservation of charge $(\partial\rho/\partial t + \nabla\cdot\underline{J} = 0)$. (14 points)

Problem 3: For a type I superconductor the magnetic field $\underline{B}$ must be identically equal to zero everywhere within the superconductor. A line current with current I_o flowing in the z-direction is located at $x=0$, $y=d>0$. The region $y>0$ is vacuum. The region $y<0$ is occupied by a type I superconductor.



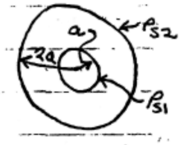
(a) What is the boundary condition on B_y at $y=0$? (6 points)

(b) By demonstrating that the boundary condition in (a) is satisfied, show that the solution for $\underline{B}$ in $y>0$ can be obtained from a problem in which the superconducting region is replaced by vacuum and a line current at $x=0$, $y=-d$, with current I_o flowing in the minus z-direction. (13 points) (10points)

(c) What is the force per unit length on the line current at $x=0$, $y=d$?

(d) What is the surface current density on the surface of the superconductor? (12 points)

Problem 4: Consider two concentric spherical surface charge densities $\rho_{s1}=4\rho_o$ and $\rho_{s2}=-\rho_o$ located in vacuum at $R=a$ and $R=2a$ respectively.



(a) Determine the electric field in the regions $R<a$, $a<R<2a$, and $R>2a$. (11 pts.)

(b) Assuming zero potential at $R=\infty$, determine $V(R)$ in $R<a$, $a<R<2a$, and $R>2a$. (12 points)

(c) What is the electrostatic stored energy of this charge configuration? (13 pts.)

Problem 5: An infinitely long, grounded, perfectly conducting cylinder of radius a is situated in vacuum. The cylinder's axis coincides with the z-axis. Far from the cylinder the electric field becomes constant: $\underline{E}$ approaches $E_o \underline{a}_y$ as r approaches infinity.

(a) Find the potential $V(r,\phi)$ in the region $r>a$. (15 points)

(b) Find the surface charge density $\rho_s(\phi)$ at $r=a$. (11 points)

SOLUTIONS TO FINAL EXAM

PROBLEM 1 (a) $\oint \underline{H}\cdot d\underline{l} = I$, $2\pi r H_\phi = I$, $H_\phi = I/2\pi r$; $\underline{B} = \mu \underline{H}$, $B_\phi = \frac{\mu_0 I}{2\pi r}$ for $a<r<2a$, $\frac{\mu_0 I}{\pi r}$ for $2a<r<3a$

(b) $\Phi = \int_a^{2a} B_\phi\, dr\, l + \int_{2a}^{3a} B_\phi\, dr\, l = \frac{\mu_0 I l}{2\pi}\left[\ln 2 + 2\ln\frac{3}{2}\right] = LI$; $L/l = \frac{\mu_0}{2\pi}\left[\ln 2 + \ln\frac{9}{4}\right]$

PROBLEM 2 (a) $\oint_{C(t)} \underline{E}'\cdot d\underline{l} = -d\Phi/dt \Rightarrow V_R = \text{voltage} = -\frac{d\Phi}{dt} = -B\frac{dA}{dt} = -B(-5\times 7)\ln 9/2$

$V_R = 3\times 35 = 105$ Volts ; $I_R = \frac{105 \text{ volts}}{13\ \Omega} = \frac{105}{13}$; $\underline{F} = \underline{a}_x BI\times 5 = \underline{a}_x \underbrace{5\times 3\times \frac{105}{13}}_{121}$ Newtons

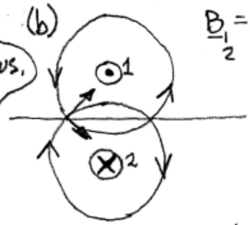
(b) $\nabla\times\underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t}$; $\nabla\cdot\nabla\times\underline{H} = \nabla\cdot\underline{J} + \partial\nabla\cdot\underline{D}/\partial t$. But $\nabla\cdot\nabla\times \equiv 0$

and $\nabla\cdot\underline{D} = \rho$. So $0 = \nabla\cdot\underline{J} + \partial\rho/\partial t$

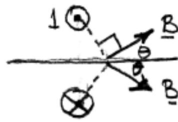
PROBLEM 3

(a) B_y is continuous. $B_y \equiv 0$ in $y<0 \Rightarrow$ $B_y = 0$ as $y\to 0$ from $y>0$.

(b)



$\underline{B}_{\substack{1\\2}} = \pm \frac{\mu_0 I_0}{2\pi r}\underline{a}_{\phi_{\substack{1\\2}}}$



$\underline{B}_1$ at $y=0$

$\underline{B}_2$ at $y=0$

the y components of these cancel

$+\frac{\mu_0 I_0}{2\pi(2d)}$

(c) Force from image is repulsive. Force/length $= \underline{a}_y I_0 \underbrace{B_x\big|_{\text{due to}}}_{2 \text{ at } 1} = \underline{a}_y \frac{\mu_0 I_0^2}{4\pi d}$

(d) $\underline{J} = J_z(x)\,\underline{a}_z = -H_x(x,y=0)\,\underline{a}_z$. From the picture above at $y=0$ the H_x due to 1 and 2 are equal and add to give twice the H_x due to one of them: $J_z = -2\frac{I_0}{2\pi\sqrt{d^2+x^2}}\cdot\underbrace{\frac{d}{\sqrt{d^2+x^2}}}_{\cos\theta} = -\frac{I_0 d}{\pi(d^2+x^2)}$

PROBLEM 4

(a) $Q_1 = 4\pi a^2(4\rho_0) = 16\pi a^2\rho_0$; $Q_2 = 4\pi(2a)^2(-\rho_0) = -16\pi a^2\rho_0$; $\underline{E} = E_R(R)\underline{a}_R$

$\oint \underline{E}\cdot d\underline{A} = Q(R)/\epsilon_0 \Rightarrow E_R = 0$ for $R<a$ and $R>2a$

$E_R(R) = Q_1/4\pi R^2\epsilon_0 = 4(a/R)^2\rho_0/\epsilon_0$ for R between a and $2a$

(b) $V(R) = -\int_\infty^R E_R(R)\,dR = -\int_{2a}^R E_R(R)\,dR = (4a^2\rho_0/\epsilon_0)\left(\frac{1}{R}-\frac{1}{2a}\right)$ for $2a>R>a$

$V = \frac{4a^2\rho_0}{\epsilon_0}\left(\frac{1}{a}-\frac{1}{2a}\right) = \frac{2a\rho_0}{\epsilon_0}$ for $R<a$; $V(R)\equiv 0$ for $R>2a$.

(c) $W_E = \frac{1}{2}\int \rho V\,dv \Rightarrow W_E = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2\cancelto{0}{V_2} = \frac{1}{2}(16\pi a^2\rho_0)\left(\frac{2a\rho_0}{\epsilon_0}\right) = 16\pi a^3\rho_0^2/\epsilon_0$

PROBLEM 5 (a) With cylinder removed $V = -E_0 y = -E_0 r\sin\phi$. With the cylinder present $V = -E_0 r\sin\phi + \tilde{V}$ where $\tilde{V}(r=a,\phi) = E_0 a\sin\phi$ in order to give $V=0$ at $r=a$.

$\nabla^2\tilde{V} = 0$ & $\tilde{V}\to 0$ as $r\to\infty \Rightarrow \tilde{V} = K\sin\phi/r$, $K = E_0 a^2$ to give correct $\tilde{V}(a,\phi)$

$V(r,\phi) = (-E_0 r + E_0 a^2/r)\sin\phi = -E_0 a\left[\frac{r}{a}-\frac{a}{r}\right]\sin\phi$.

(b) $\rho_s(\phi) = \epsilon_0 E_r(r=a,\phi) = -\epsilon_0 \partial V/\partial r\big|_{r=a} = \epsilon_0\left\{E_0\left(1+\frac{a^2}{r^2}\right)\sin\phi\right\}_{r=a} = 2\epsilon_0 E_0\sin\phi$

Spring '09: You are not responsible for the material covered in this ↑ problem.