PRACTICE EXAM

EXAM I PROF. E. OTT

NOTE: Be sure to show all work and derivations. An answer without supporting work is not sufficient.

Problem 1 (60 points)

The electric field in a vacuum region of space is given by

$$\mathbf{E} = (3y\hat{y} + z\hat{z}) \quad (volts/m.)$$

(a) How much work does an external agent do to move a point charge of $10^{-6}C$. from the point $x = 0, y = 2m., z = -1m.$ to the point $x = 0, y = 2m., z = 0$?

(b) What is the volume charge density $\rho(x, y, z)$ necessary to set up this electric field?

(c) Using the surface integral method, calculate the total charge enclosed by the cubical surface whose six sides lie on the planes, $x = 0, x = 1m., y = 0, y = 1m., z = 0$, and $z = 1m$.

Problem 2 (20 points)

There is a point charge of Q coulombs located in vacuum at the origin of a spherical coordinate system. In addition a uniform surface charge density of σ coulombs/$m.^2$ exists on the surface $r = a$ (see blackboard). Find the electric field $\mathbf{E}$ in the regions $0 < r < a$ and $a < r$

Problem 3 (20 points)

The region $x < 0$ is vacuum. The region $x > 0$ is filled with a dielectric of dielectric constant $\epsilon = 2\epsilon_o$. There is a uniform electric field

$$\mathbf{E} = (5\hat{x} + 6\hat{y}) \quad volts/m.$$

throughout the region $x < 0$. What is $\mathbf{E}$ in $x > 0$? (There is no free surface charge density on the dielectric surface at $x = 0$.)

PRACTICE

SOLUTIONS TO EXAM I

Prob. 1 (a) $W = -q\int_a^b \underline{E}\cdot d\underline{l} = -q\int_{-1}^{0} z\,dz = -q\,\frac{z^2}{2}\Big]_{-1}^{0} = +\frac{q}{2} = \boxed{5\times10^{-7}\,J}$

$d\underline{l} = \hat{z}\,dz$

(20 pts.)

(b) $\rho = \epsilon_0 \nabla\cdot\underline{E} = \epsilon_0\left(\frac{\partial}{\partial y}3y + \frac{\partial}{\partial z}z\right) = \epsilon_0(3+1) = \boxed{4\epsilon_0}$

(20 pts.)

(c) $Q_{enc} = \epsilon_0 \oint \underline{E}\cdot d\underline{a}$. $E_x = 0$, $E_y = 0$ at $y=0$, $E_z = 0$ at $z=0$

$E_y = 3$ at $y=1$, $E_z = 1$ at $z=1$. Thus

$\oint \underline{E}\cdot d\underline{a} = \int_0^1\int_0^1 \underbrace{E_y}_{3}\,dx\,dz + \int_0^1\int_0^1 \underbrace{E_z}_{1}\,dx\,dy = 3+1=4 \Rightarrow \boxed{Q_{enc} = 4\epsilon_0}$

(20 pts)

Prob. 2 (20 pts.)

$\oint \underline{E}\cdot d\underline{a} = Q_{enc}/\epsilon_0 \Rightarrow 4\pi r^2 E_r(r) = Q_{enc}/\epsilon_0 \Rightarrow E_r = \frac{Q_{enc}}{4\pi\epsilon_0 r^2}$

For $r<a$, $Q_{enc} = Q \Rightarrow \boxed{E_r = Q/(4\pi\epsilon_0 r^2)}$

For $r>a$, $Q_{enc} = Q + 4\pi a^2\sigma \Rightarrow \boxed{E_r = \frac{Q}{4\pi\epsilon_0 r^2} + \frac{\sigma}{\epsilon_0}\left(\frac{a}{r}\right)^2}$

Prob. 3 (20 pts.)

$E^{\parallel} = E_y$ is continuous. $\Downarrow$ $E_y = 6$ on both sides

$D^{\perp} = D_x$ is continuous $\Downarrow$ $\epsilon_0 \underbrace{E_x(x<0)}_{5} = 2\epsilon_0 E_x(x>0)$ $\Downarrow$ $E_x(x>0) = 5/2$

$\boxed{\underline{E} = \frac{5}{2}\hat{x} + 6\hat{y}}$ in $x>0$