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APPLICATION BULLETIN

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NOISE ANALYSIS OF FET TRANSIMPEDANCE AMPLIFIERS

The availability of detailed noise spectral density characteristics for the OPA111 amplifier allows an accurate noise error analysis in a variety of different circuit configurations. The fact that the spectral characteristics are guaranteed maximums allows absolute noise errors to be truly bounded. Other FET amplifiers normally use simpler specifications of rms noise in a given bandwidth (typically 10Hz to 10kHz) and peak-to-peak noise (typically specified in the band 0.1Hz to 10Hz). These specifications do not contain enough information to allow accurate analysis of noise behavior in any but the simplest of circuit configurations.

Noise in the OPA111 can be modeled as shown in Figure 1. This model is the same form as the DC model for offset voltage (E_{OS}) and bias currents (I_B). In fact, if the voltage $e_n(t)$ and currents $i_n(t)$ are thought of as general instantaneous error sources, then they could represent either noise or DC offsets. The error equations for the general instantaneous model are shown in Figure 2.

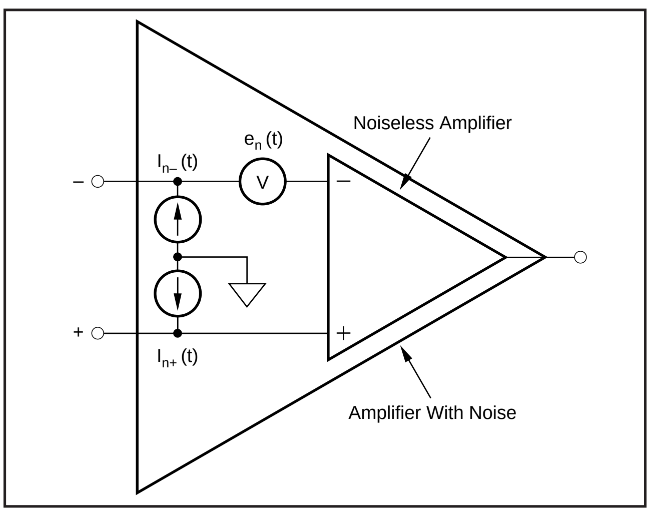

FIGURE 1. Noise Model of OPA111.

If the instantaneous terms represent DC errors (i.e., offset voltage and bias currents) the equation is a useful tool to compute actual errors. It is not, however, useful in the same *direct* way to computer noise errors. The basic problem is that noise cannot be predicted as a function of time. It is a random variable and must be described in probabilistic terms. It is normally described by some type of average—most commonly the rms value.

$$N\ \text{rms} \overset{\Delta}{=} \sqrt{1/T\int_0^T n^2(t)\,dt} \qquad (1)$$

where N rms is the rms value of some random variable n(t). In the case of amplifier noise, n(t) represents either $e_n(t)$ or $i_n(t)$.

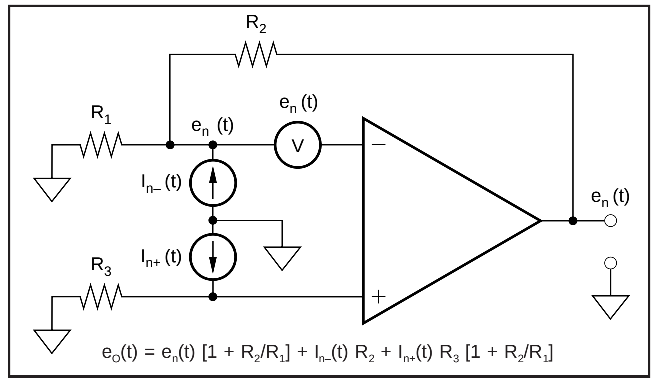

FIGURE 2. Circuit With Error Sources.

The internal noise sources in operational amplifiers are normally uncorrelated. That is, they are randomly related to each other in time and there is no systematic phase relationship. Uncorrelated noise quantities are combined as root-sum-squares. Thus, if $n_1(t)$, $n_2(t)$, and $n_3(t)$ are uncorrelated then their combined value is

$$N_{TOTAL}\ \text{rms} = \sqrt{N_1^2\ \text{rms} + N_2^2\text{rms} + N_3^2\ \text{rms}} \qquad (2)$$

The basic approach in noise error calculations then is to identify the noise sources, segment them into conveniently handled groups (in terms of the shape of their noise spectral densities), compute the rms value of each group, and then combine them by root-sum-squares to get the total noise.

TYPICAL APPLICATION

The circuit in Figure 3 is a common application of a low noise FET amplifier. It will be used to demonstrate the above noise calculation method.

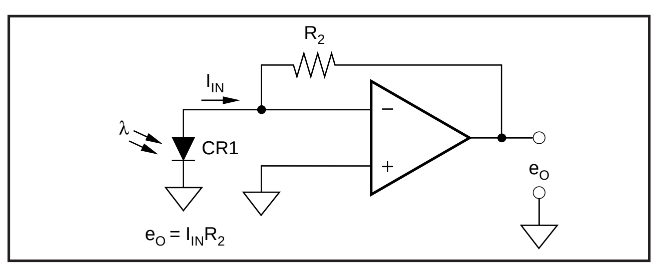

FIGURE 3. Pin Photo Diode Application.

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CR1 is a PIN photodiode connected in the photovoltaic mode (no bias voltage) which produces an output current i_{IN} when exposed to the light, λ.

A more complete circuit is shown in Figure 4. The values shown for C_1 and R_1 are typical for small geometry PIN diodes with sensitivities in the range of 0.5 A/W. The value of C_2 is what would be expected from stray capacitance with moderately careful layout (0.5pF to 2pF). A larger value of C_2 would normally be used to limit the bandwidth and reduce the voltage noise at higher frequencies.

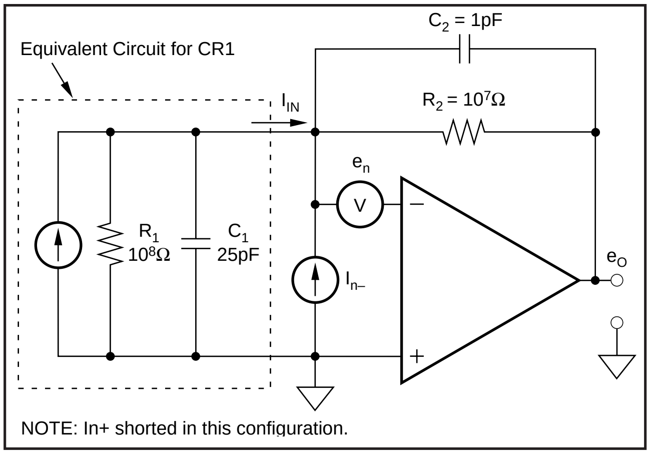


FIGURE 4. Noise Model of Photodiode Application.

In Figure 4, e_n and i_n represent the amplifier's voltage and current spectral densities, $e_n(\omega)$ and $i_n(\omega)$, respectively. These are shown in Figure 5.

Figure 6 shows the desired "gain" of the circuit (transimpedance of $e_O/i_{IN} = Z_2(s)$). It has a single-pole rolloff at $f_2 = 1/(2\pi R_2 C_2) = \omega_2/2\pi$. Output noise is minimized if f_2 is made smaller. Normally R_2 is chosen for the desired DC transimpedance based on the full scale input current (i_{IN} full scale) and maximum output (e_O max). Then C_2 is chosen to make f_2 as small as possible consistent with the necessary signal frequency response.

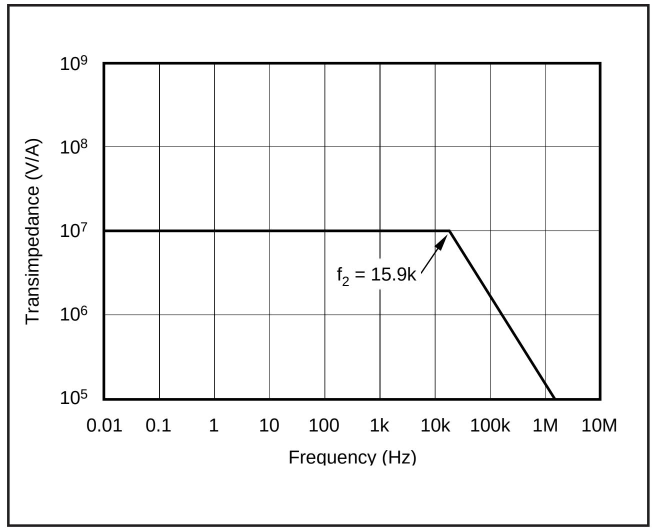


FIGURE 6. Transimpedance.

VOLTAGE NOISE

Figure 7 shows the noise voltage gain for the circuit in Figure 4. It is derived from the equation

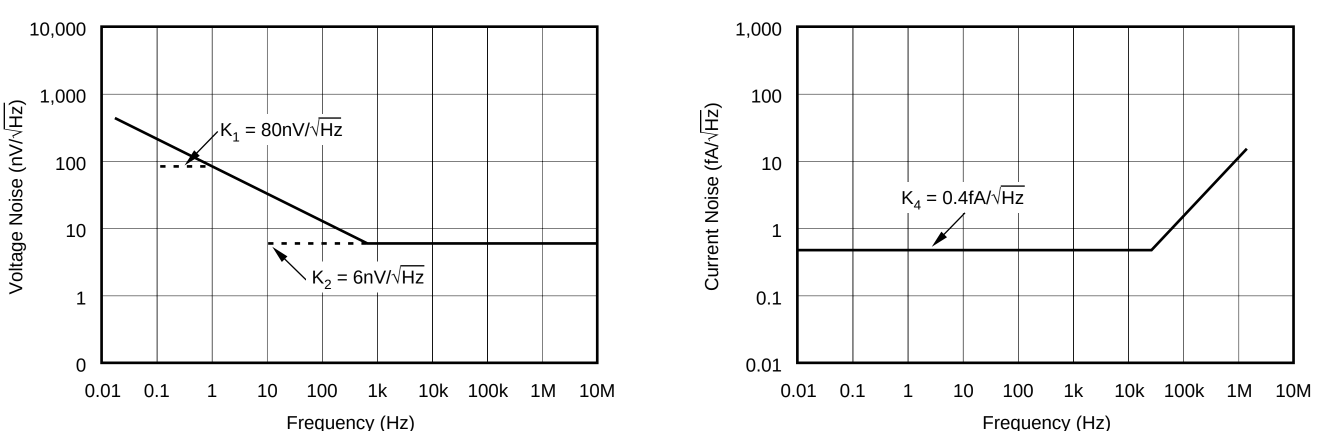
$$e_O = e_n\left[\frac{A}{1+A\beta}\right] = e_n\frac{1}{\beta}\left[\frac{1}{1+\frac{1}{A\beta}}\right] \qquad (3)$$

where:

- A = $A(\omega)$ is the open-loop gain.
- β = $\beta(\omega)$ is the feedback factor. It is the amount of output voltage feedback to the input of the op amp.
- $A\beta$ = $A(\omega)\,\beta(\omega)$ is the loop gain. It is the amount of the output voltage feedback to the input and then amplified and returned to the output.

Note that for large loop gain ($A\beta >> 1$)

$$e_O \cong e_n\frac{1}{\beta} \qquad (4)$$



(a) Voltage Noise (b) Current Noise

FIGURE 5. Noise Voltage and Current Spectral Density.

For the circuit in Figure 4 it can be shown that

$$\frac{1}{\beta} = 1 + \frac{R_2\left(R_1C_{1S}+1\right)}{R_1\left(R_2C_{2S}+1\right)} \tag{5}$$

This may be rearranged to

$$\frac{1}{\beta} = \frac{R_2+R_1}{R_1'}\left[\frac{\tau_A s+1}{\tau_2 s+1}\right] \tag{5a}$$

where $\tau a = (R_1 \parallel R_2)\ (C_1 \parallel C_2)$

$$\frac{1}{\beta} = \left[\frac{R_1R_2}{R_1+R_2}\right](C_1+C_2) \tag{5b}$$

and $\tau_2 = R_2C_2$. (5c)

$$\text{Then, } f_a = \frac{1}{2\pi\ \tau_a} \text{ and } f_2 = \frac{1}{2\pi\ \tau_2} \tag{5d}$$

For very low frequencies ($f << f_a$), s approaches zero and equation 5 becomes

$$\frac{1}{\beta} = 1 + \frac{R_2}{R_1}. \tag{6}$$

For very high frequencies ($f >> f_2$), s approaches infinity and equation 5 becomes

$$\frac{1}{\beta} = 1 + \frac{C_1}{C_2}. \tag{7}$$

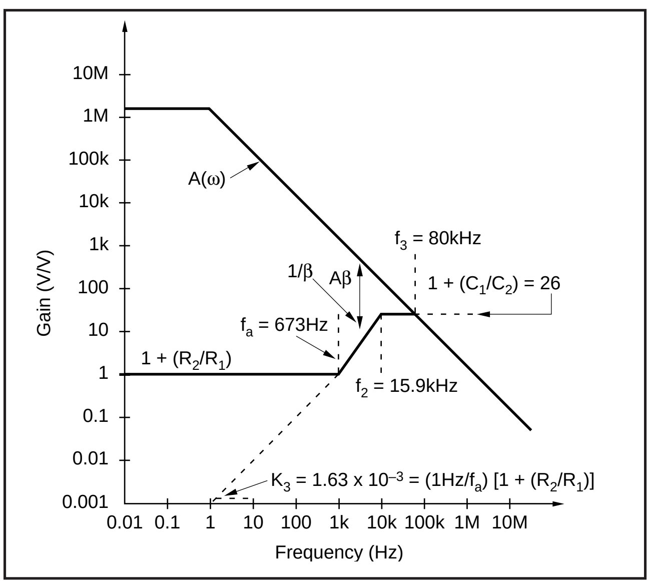


FIGURE 7. Noise Voltage Gain.

The noise voltage spectral density at the output is obtained by multiplying the amplifier's noise voltage spectral density (Figure 5a) times the circuits noise gain (Figure 7). Since both curves are plotted on log-log scales, the multiplication can be performed by the addition of the two curves. The result is shown in Figure 8.

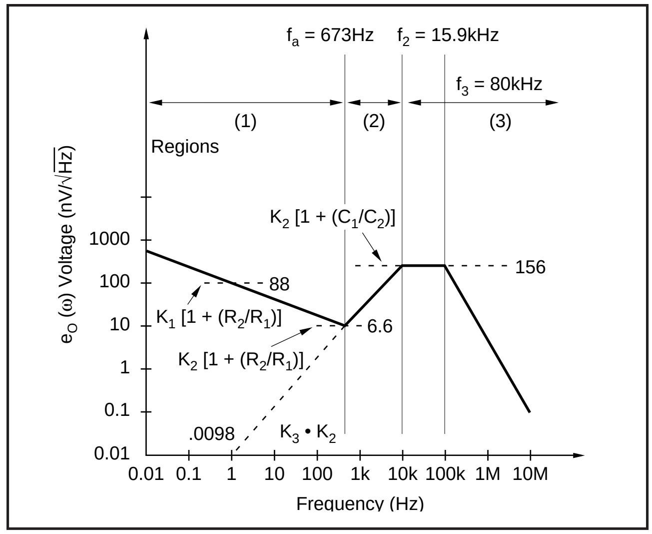


FIGURE 8. Output Voltage Noise Spectral Density.

The total rms noise at the amplifier's output due to the amplifier's internal voltage noise is derived from the $e_O(\omega)$ function in Figure 8 with the following expression:

$$E_O \text{ rms} = \sqrt{\int_{-\infty}^{+\infty} e_O{}^2(\omega)\, d\omega} \tag{8}$$

It is both convenient and informative to calculate the rms noise using a piecewise approach (region-by-region) for each of the three regions indicated in Figure 8.

Region 1; f_1 = 0.01Hz to f_C = 100Hz

$$E_{n1} \text{ rms} = K_1\left[1+\frac{R_2}{R_1}\right]\sqrt{\ell n\left(\frac{f_C}{f_1}\right)} \tag{9}$$

$$= 80\text{nV}/\sqrt{\text{Hz}}\left[1+\frac{10^7}{10^8}\right]\sqrt{\ell n\left(\frac{100}{0.01}\right)} \tag{9a}$$

$$= 0.267\mu\text{V}$$

This region has the characteristic of 1/f or "pink" noise (slope of –10dB per decade on the log-log plot of $e_n\omega$). The selection of 0.01Hz is somewhat arbitrary but it can be shown that for this example there would be only negligible additional contribution by extending f_1 several decades lower. Note that $K_1 (l + R_2/R_1)$ is the value of e_O at f = 1Hz.

Region 2; f_a = 673Hz to f_2 = 15.9kHz

$$E_{n2} \text{ rms} = K_2 \bullet K_3\sqrt{\frac{f_2{}^3}{3} - \frac{f_a{}^3}{3}} \tag{10}$$

$$= \left(6\text{nV}/\sqrt{\text{Hz}}\right)(1.63\times10^{-3})\sqrt{\frac{(15.9\text{kHz})^3}{3} - \frac{(673)^3}{3}} \tag{10a}$$

$$= 11.3\mu\text{V} \tag{9}$$

This is the region of increasing noise gain (slope of +20dB/decade on the log-log plot) caused by the lead network formed by the resistance $R_1 \parallel R_2$ and the capacitance ($C_1 + C_2$). Note that $K_3 \cdot K_2$ is the value of the e_O (ω) function for this segment projected back to 1Hz.

Region 3; f > 15.9kHz

$$E_{n3}\ \text{rms} = K_2\left(1+\frac{C_1}{C_2}\right)\sqrt{\left(\frac{\pi}{2}\right)f_3 - f_2} \quad (11)$$

(11a)

$$= \left(6\text{nV}/\sqrt{\text{Hz}}\right)\left(1+\frac{25}{1}\right)\sqrt{\left(\frac{\pi}{2}\right)(80\text{k}) - 15.9\text{k}}$$

$$= 51.7\mu\text{V}$$

This is a region of white noise with a single order rolloff at f_3 = 80kHz caused by the intersection of the 1/β curve and the open-loop gain curve. The value of 80kHz is obtained from observing the intersection point of Figure 7. The π/2 applied to f_3 is to convert from a 3dB corner frequency to an effective noise bandwidth.

CURRENT NOISE

The output voltage component due to current noise is equal to:

$$E_{ni} = i_n \times Z_2(s) \quad (12)$$

where $Z_2(s) = R_2 \parallel X_{C2}$ (12a)

This voltage may be obtained by combining the information from Figures 5 (b) and 6 together with the open loop gain curve of Figure 7. The result is shown in Figure 9.

Using the same techniques that were used for the voltage noise:

Region l; 0.1Hz to 10kHz

$$E_{ni1} = 4\times10^{-9}\sqrt{10\text{k} - 0.1} \quad (13)$$

$$= 0.4\mu\text{V}$$

Region 2; 10kHz to 15.9kHz

$$E_{ni2} = 4\times10^{-13}\sqrt{\frac{(15.9\text{kHz})^3}{3} - \frac{(10\text{kHz})^3}{3}}$$

$$= 0.4\mu\text{V} \quad (13a)$$

Region 3; f > 15.9kHz

$$E_{ni3} = 6.36\times10^{-9}\sqrt{\frac{\pi}{2}(80\text{kHz}) - 15.9\text{kHz}}$$

$$= 2.1\mu\text{V} \quad (13b)$$

$$E_{ni\ TOTAL} = 10^{-6}\sqrt{(0.4)^2 + (0.4)^2 + (2.1)^2}$$

$$= 2.2\mu\text{Vrms} \quad (13c)$$

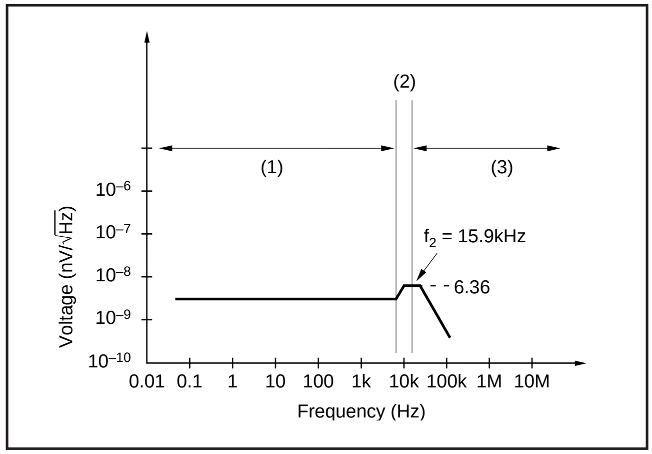


FIGURE 9. Output Voltage Due to Noise Current.

RESISTOR NOISE

For a complete noise analysis of the circuit in Figure 4, the noise of the feedback resistor, R_2, must also be included. The thermal noise of the resistor is given by:

$$E_R\ \text{rms} = \sqrt{4kTRB} \quad (14)$$

K = Boltzmann’s constant = 1.38×10^{-23} Joules/°Kelvin
T = Absolute temperature (°K)
R = Resistance (Ω)
B = Effective noise bandwidth (Hz) (ideal filter assumed)

At 25°C this becomes

$$E_R\ \text{rms} \cong 0.13\sqrt{RB}$$

E_R rms in µV
R in MΩ
B in Hz

For the circuit in Figure 4

$$R_2 = 10^7\Omega = 10\text{M}\Omega$$

$$B = \frac{\pi}{2}(f_2) = \frac{\pi}{2}15.9\text{kHz}$$

Then

$$E_R\ \text{rms} = \left(411\text{nV}/\sqrt{\text{Hz}}\right)\sqrt{B}$$

$$= \left(411\text{nV}/\sqrt{\text{Hz}}\right)\sqrt{\frac{\pi}{2}15.9\text{kHz}}$$

$$= 64.9\mu\text{Vrms}$$

TOTAL NOISE

The total noise may now be computed from

$$E_{n\,TOTAL} = \sqrt{E_{n1}{}^2 + E_{n2}{}^2 + E_{n3}{}^2 + E_{nR}{}^2 + E_{ni}{}^2} \qquad (15)$$

$$= 10^{-6}\sqrt{(0.293)^2 + (11.3)^2 + (51.7)^2 + (64.9)^2 + (2.2)^2} \qquad (15a)$$

$$= 83.8\mu Vrms$$

CONCLUSIONS

Examination of the results in equation (16b) together with the curves in Figure 8 leads to some interesting conclusions.

The largest component is the resistor noise E_{nR} (60% of the total noise). A lower resistor value decreases resistor noise as a function of √R, but it also lowers the desired signal gain as a direct function of R. Thus, lowering R reduces the signal-to-noise ratio at the output which shows that the feedback resistor should be as large as possible. The noise contribution due to R_2 can be decreased by raising the value of C_2 (lowering f_2) but this reduces signal bandwidth.

The second largest component of total noise comes from $E_n{}^3$ (38%). Decreasing C_1 will also lower the term $K_2(1 + C_1/C_2)$. In this case, f_2 will stay fixed and f_a will move to the right (i.e., the +20dB/decade slope segment will move to the right). This can have a significant reduction on noise without lowering the signal bandwidth. This points out the importance of maintaining low capacitance at the amplifier's input in low noise applications.

It should be noted that increasing C_2 will also lower the value of K_2 $(1 + C_1/C_2)$, and the value of f_2 (see equation 5b). This reduces signal bandwidth and the final value of C_2 is normally a compromise between noise gain and necessary signal bandwidth.

It is interesting to note that the current noise of the amplifier accounted for only 0.1% of the total E_n. This is different than would be expected when comparing the current and voltage spectral densities with the size of the feedback resistor. For example, if we define a characteristic value of resistance as

$$R_{CHARACTERISTIC} = \frac{e_n(\omega)}{i_n(\omega)} \text{ at } f = 10kHz$$

$$= \frac{6nV/\sqrt{Hz}}{0.4fA\sqrt{Hz}}$$

$$= 15M\Omega$$

Thus, in simple transimpedance circuits with feedback resistors greater than the characteristic value, the amplifier's current noise would cause more output noise than the amplifier's voltage noise. Based on this and the 10MΩ feedback resistor in the example, the amplifier noise current would be expected to have a higher contribution than the noise voltage. The reason it does not in the example of Figure 4 is that the noise voltage has high gain at higher frequencies (Figure 7) and the noise current does not (Figure 6).