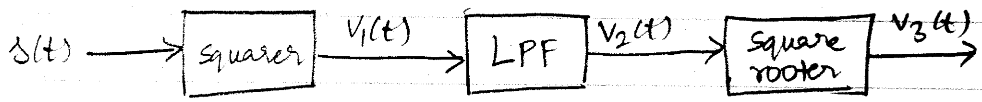
Homework 5 Solutions

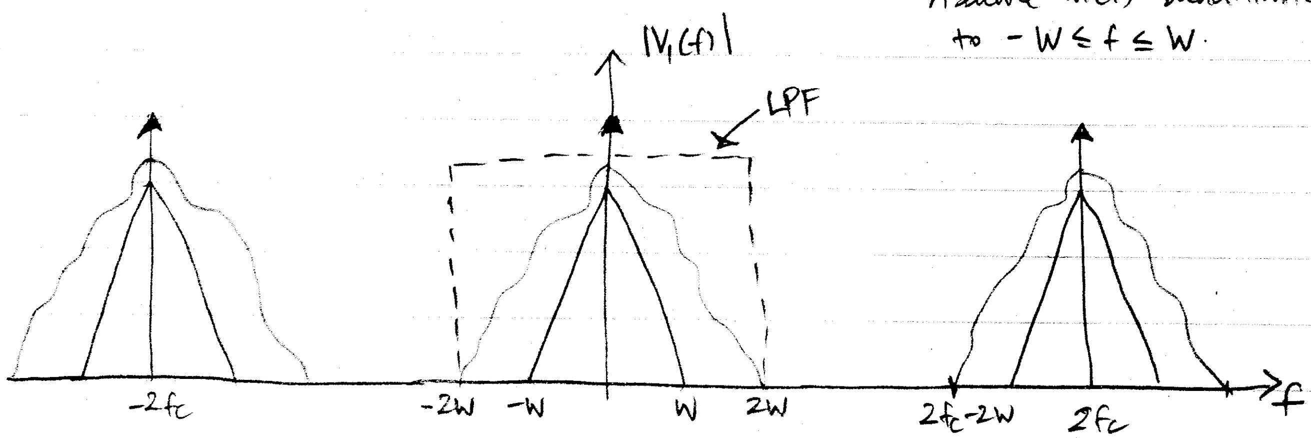
2.7



$$s(t) = A_c(1 + k_a m(t))\cos(2\pi f_c t) \quad , \quad |k_a m(t)| < 1 \quad \forall t$$

$$V_1(t) = s^2(t) = \frac{A_c^2}{2}(1 + k_a^2 m^2(t) + 2k_a m(t))\,(1 + \cos(4\pi f_c t))$$

Assume m(t) band-limited to $-W \le f \le W$.



Use LPF with cut-off frequency 2W.

If $2f_c - 2W > 2W$, i.e., $f_c > 2W$, then the output of the LPF is

$$V_2(t) = \frac{A_c^2}{2}(1 + k_a m(t))^2$$

$$V_3(t) = \sqrt{V_2(t)} = \frac{A_c}{\sqrt{2}}(1 + k_a m(t))$$

Since the parameters A_c and k_a are known, m(t) can be recovered.

2.12

$s(t) = A_c m_1(t) \cos(2\pi f_c t) + A_c m_2(t) \sin(2\pi f_c t)$ ← transmitted

$$S(f) = \frac{A_c}{2}\left[M_1(f-f_c) + M_1(f+f_c)\right] + \frac{A_c}{2j}\left[M_2(f-f_c) - M_2(f+f_c)\right].$$

Let $r(t)$ denote the received signal. Then $R(f) = \mathcal{F}(r(t))$ is

$$R(f) = H(f)\, S(f)$$
$$= \frac{A_c}{2} H(f)\left[M_1(f-f_c) + M_1(f+f_c) + \frac{1}{j} M_2(f-f_c) - \frac{1}{j} M_2(f+f_c)\right]$$

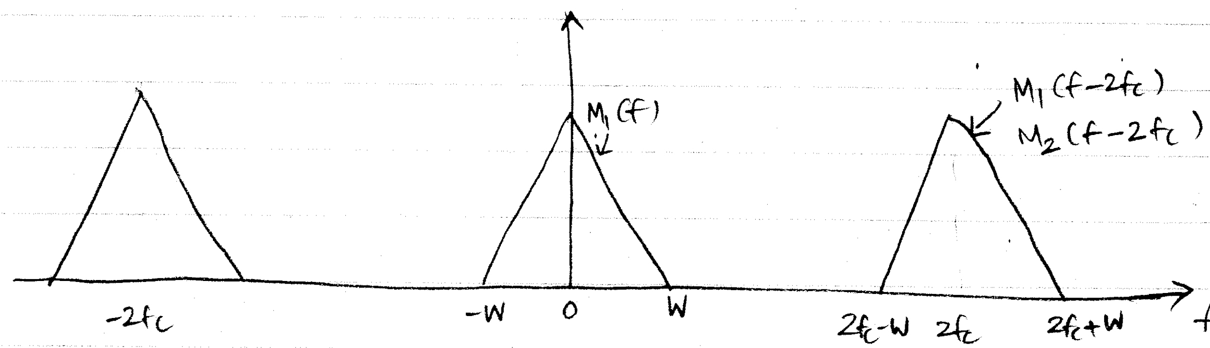
Denote output of the product modulators (at receiver) by $y_1(t)$ and $y_2(t)$ resp.

$$y_1(t) = r(t)\; 2\cos(2\pi f_c t)$$
$$Y_1(f) = R(f-f_c) + R(f+f_c)$$
$$= \frac{A_c}{2} H(f-f_c)\left[M_1(f-2f_c) + M_1(f) + \frac{1}{j} M_2(f-2f_c) - \frac{1}{j} M_2(f)\right]$$
$$+ \frac{A_c}{2} H(f+f_c)\left[M_1(f) + M_1(f+2f_c) + \frac{1}{j} M_2(f) - \frac{1}{j} M_2(f+2f_c)\right]$$

If $H(f_c+f) = H^*(f_c-f)\;,\; 0 \le f \le W$

i.e., $H(f_c+f) = H(f-f_c)\;,\; 0 \le f \le W$ ($\because$ for real-valued $x(t)$, $X(-f) = X^*(f)$)

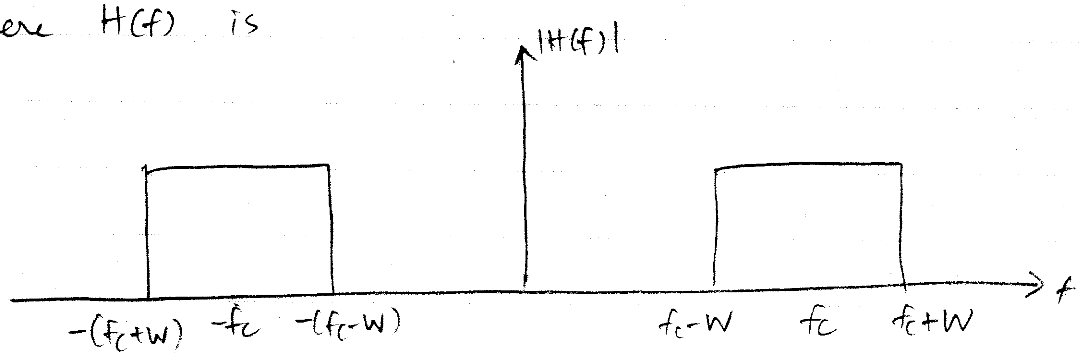
$$\therefore\ Y_1(f) = A_c H(f-f_c) M_1(f) + \frac{A_c}{2} H(f-f_c)\Big[M_1(f-2f_c) + M_1(f+2f_c) + \frac{1}{j} M_2(f-2f_c) - \frac{1}{j} M_2(f+2f_c)\Big]$$



If $m_1(t)$ and $m_2(t)$ are band-limited to $-W \le f \le W$ and $2f_c - W \ge W$, i.e., $f_c \ge W$, then output of LPF is

$$A_c H(f-f_c) M_1(f)$$

If channel response $H(f)$ is known, or in the typical case where $H(f)$ is



the message $m_1(t)$ can be recovered.

Similarly, $m_2(t)$ can be recovered from $y_2(t)$ in the second branch.

2.13 Local carrier (at receiver) has phase error ϕ.

With same notation as in problem 2.12, now

$$y_1(t) = r(t) \cdot 2\cos(2\pi f_c t + \phi)$$
$$= 2r(t)\left[\cos 2\pi f_c t \cos\phi - \sin 2\pi f_c t \sin\phi\right]$$

In problem 2.12, the output of LPF for input $2r(t)\cos 2\pi f_c t$ was

$$A_c H(f-f_c) M_1(f)$$

and similarly, the output of LPF (in the second branch) for input $2r(t)\sin 2\pi f_c t$ was

$$A_c H(f-f_c) M_2(f).$$

$\therefore$ for the $y_1(t)$ in this problem (with phase error ϕ), output of first LPF is

$$A_c H(f-f_c)\left[\cos\phi\, M_1(f) - \sin\phi\, M_2(f)\right].$$

$\therefore$ cross-talk occurs!

2.15

$$m(t) = \frac{1}{1+t^2}$$

(a) Amplitude mod. with 50% modulation.

$$|k_a m(t)|_{max} \times 100 = 50$$

$$\therefore \quad k_a = 0.5$$

$$s_{AM}(t) = A_c\left(1 + \frac{0.5}{1+t^2}\right)\cos(2\pi f_c t)$$

(b) DSB-SC

$$s_{DSB-SC}(t) = A_c\, m(t)\cos(2\pi f_c t)$$

$$= \frac{A_c}{1+t^2}\cos(2\pi f_c t)$$

(c) SSB - upper sideband

[From Table 2.1, P94. Also, see Prob. 2.16]

$$s_{SSB-U}(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\right]$$

$\hat{m}(t)$ = Hilbert transform of $m(t) = \frac{1}{1+t^2}$

$$= \frac{t}{1+t^2} \quad \text{[From Table A6.4, P 765]}$$

$$s_{SSB-U}(t) = \frac{A_c}{2}\left[\frac{1}{1+t^2}\cos(2\pi f_c t) - \frac{t}{1+t^2}\sin(2\pi f_c t)\right]$$

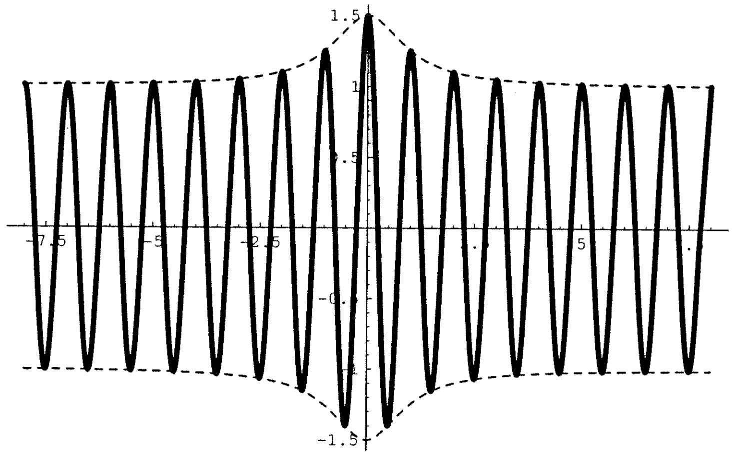
(d) SSB - lower sideband

$$s_{SSB-L}(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\right]$$

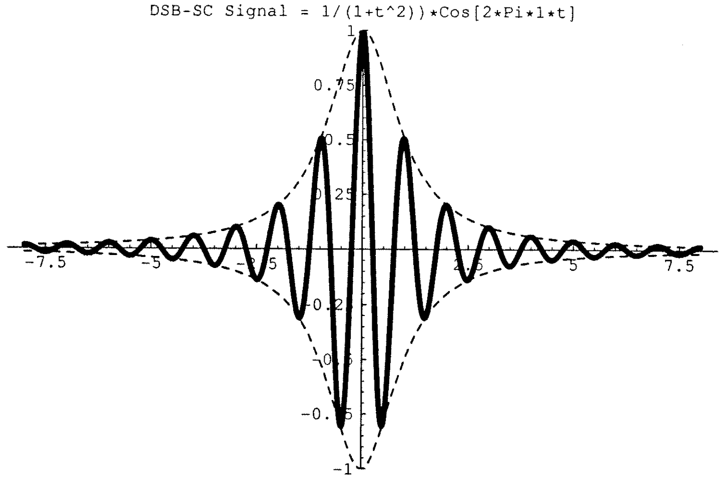
[From Table 2.1, P94. Also, see Prob. 2.16]

$$= \frac{A_c}{2}\left[\frac{1}{1+t^2}\cos(2\pi f_c t) + \frac{t}{1+t^2}\sin(2\pi f_c t)\right]$$

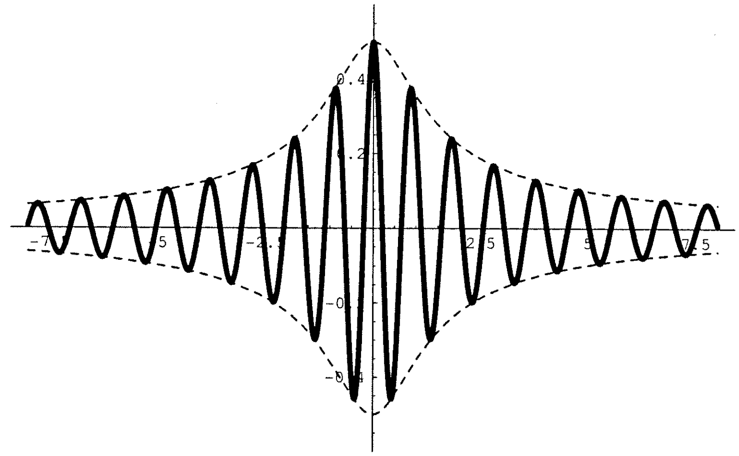
AM Signal = (1+.5/(1+t^2))*Cos[2*Pi*1*t]



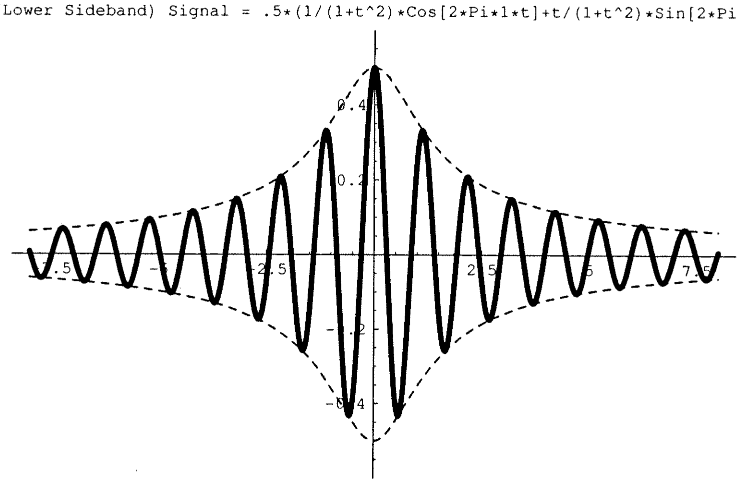
DSB-SC Signal = 1/(1+t^2))*Cos[2*Pi*1*t]



SSB (Upper Sideband) Signal = .5*(1/(1+t^2)*Cos[2*Pi*1*t]-t/(1+t^2)*Sin[2*Pi*1*t])



SSB (Lower Sideband) Signal = .5*(1/(1+t^2)*Cos[2*Pi*1*t]+t/(1+t^2)*Sin[2*Pi*1*t])



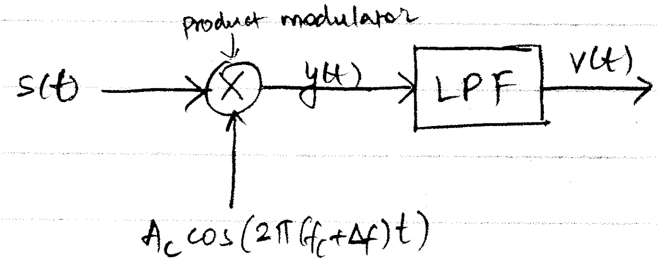
[Table 2.1, P44 and Prob. 2.16]

2.17

For upper/lower sideband,

$$s(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) \mp \hat{m}(t)\sin(2\pi f_c t)\right]$$

upper SB ($-$), lower SB ($+$)



← Demodulation

$$y(t) = s(t)\cdot A_c\cos(2\pi(f_c+\Delta f)t)$$
$$= s(t)\, A_c\cdot\left[\cos(2\pi f_c)t\cos(2\pi\Delta f t) - \sin(2\pi f_c t)\sin(2\pi\Delta f t)\right]$$

The calculations are similar to Prob. 2.12 (with $H(f)$ ignored or taken $H(f)=1$ as there is no channel here).

and $m_1(t) = m(t)$, $m_2(t) = \hat{m}(t)$.

$$S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] \mp \frac{A_c}{2j}\left[\hat{M}(f-f_c) - \hat{M}(f+f_c)\right]$$

$$Y(f) = \frac{A_c}{2}\left[S(f-(f_c+\Delta f)) + S(f+(f_c+\Delta f))\right]$$
$$= \frac{A_c^2}{4}\left[M(f-(2f_c+\Delta f)) + M(f-\Delta f) \mp \frac{1}{j}\hat{M}(f-(2f_c+\Delta f)) \pm \frac{1}{j}\hat{M}(f-\Delta f)\right]$$
$$+ \frac{A_c^2}{4}\left[M(f+\Delta f) + M(f+(2f_c+\Delta f)) \mp \frac{1}{j}\hat{M}(f+\Delta f) \pm \frac{1}{j}\hat{M}(f+(2f_c+\Delta f))\right]$$

Output of LPF :

$$V(f) = \frac{A_c^2}{4}\left[M(f-\Delta f) + M(f+\Delta f) \pm \frac{1}{j}\hat{M}(f-\Delta f) \mp \frac{1}{j}\hat{M}(f+\Delta f) \right]$$

$$= \frac{A_c^2}{4}\left[M(f-\Delta f) + M(f+\Delta f) \mp \operatorname{sgn}(f)\, M(f-\Delta f) \pm \operatorname{sgn}(f)\, M(f+\Delta f) \right]$$

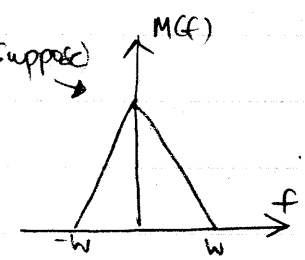
For upper sideband SSB :

$$V(f) = \begin{cases} \frac{A_c^2}{2} M(f+\Delta f) & , f \geq 0 \\ \frac{A_c^2}{2} M(f-\Delta f) & , f < 0 \end{cases}$$

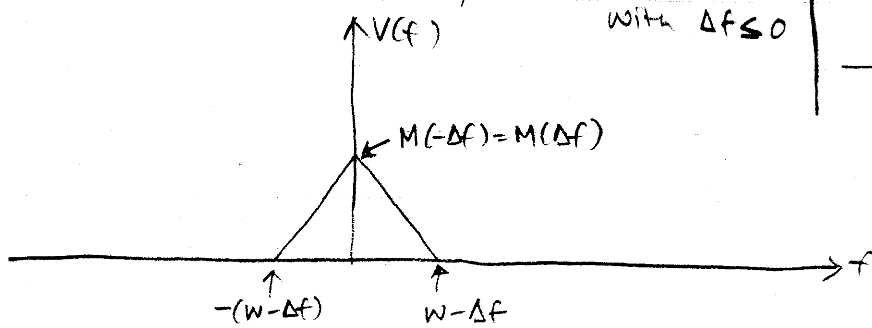
For lower sideband SSB :

$$V(f) = \begin{cases} \frac{A_c^2}{2} M(f-\Delta f) & , f \geq 0 \\ \frac{A_c^2}{2} M(f+\Delta f) & , f < 0 \end{cases}$$

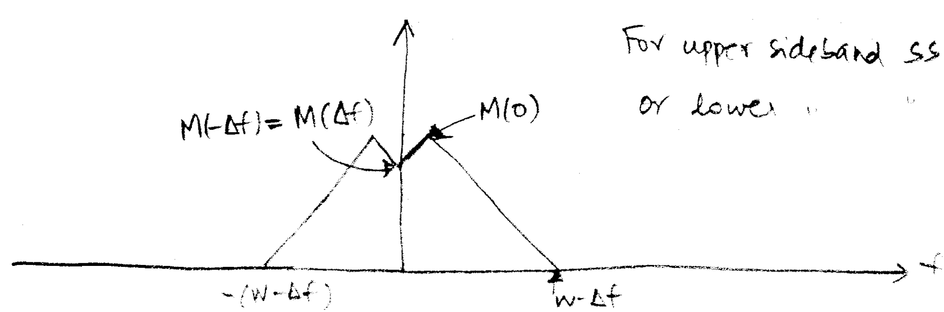
(Suppose)



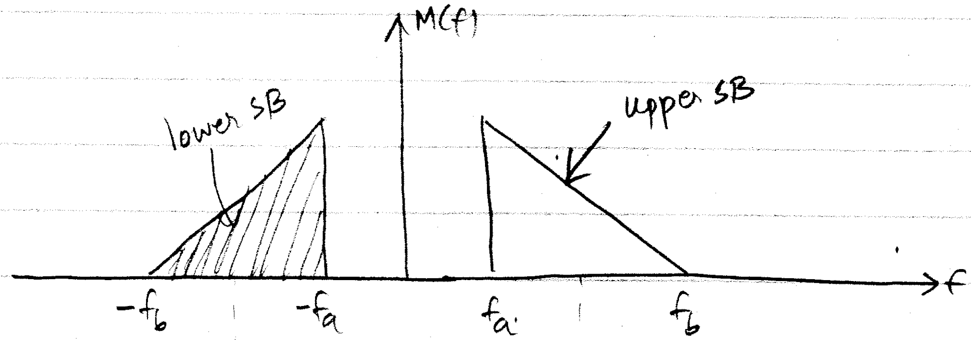
① For upper sideband SSB with $\Delta f \geq 0$, or lower sideband with $\Delta f \leq 0$



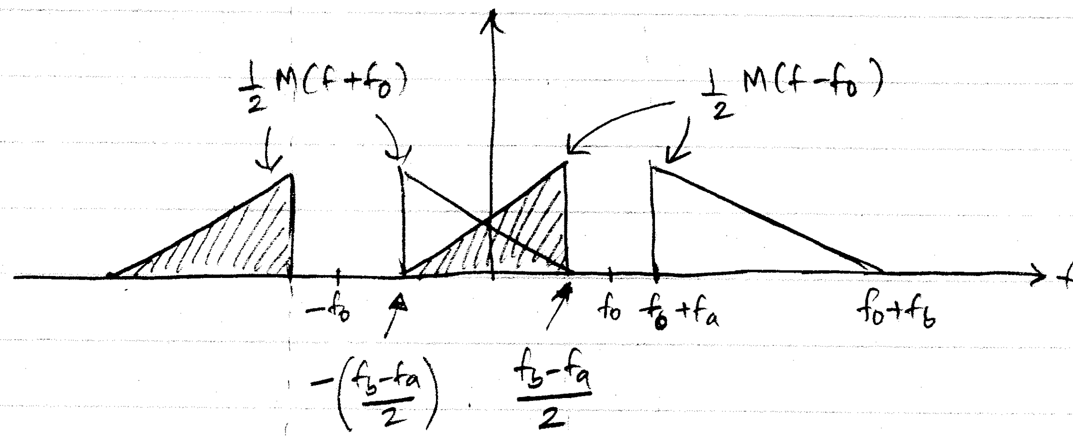
② For upper sideband SSB with $\Delta f \leq 0$ or lower " " " $\Delta f \geq 0$



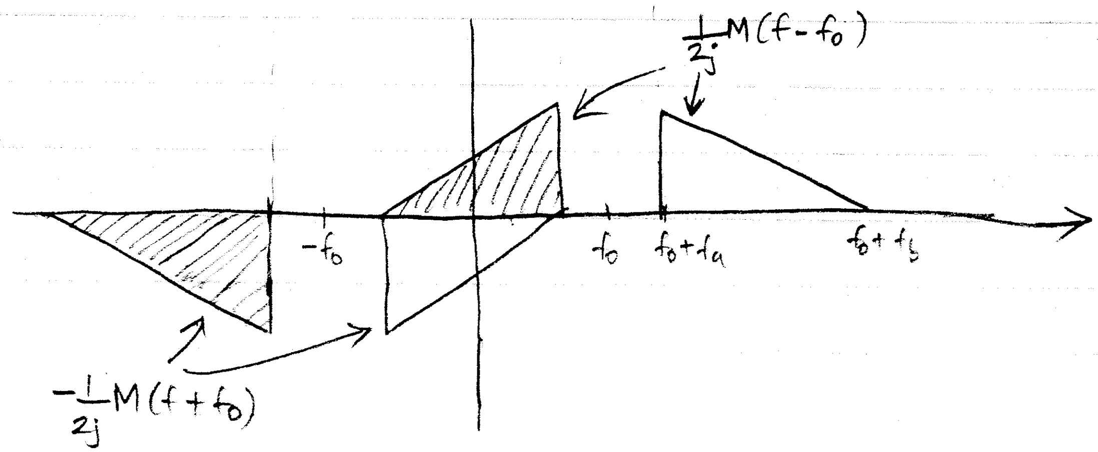
2.18 Suppose $M(f)$ have the following spectrum.



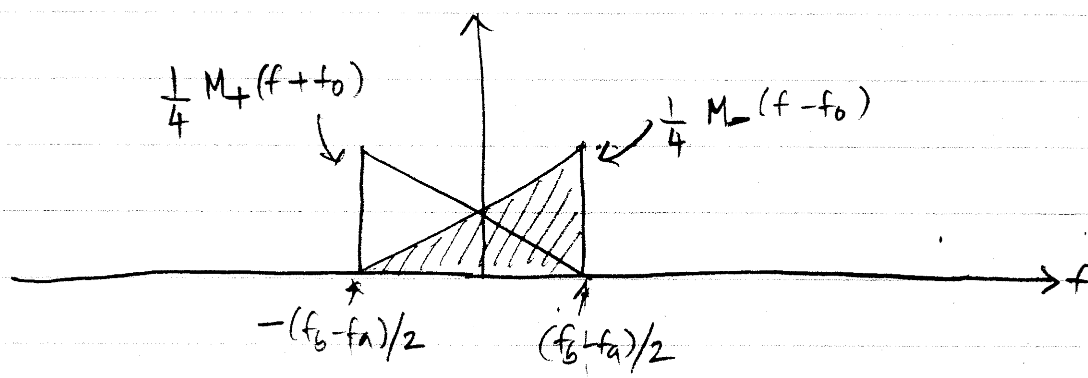
Output of first product mod. (in-phase channel)



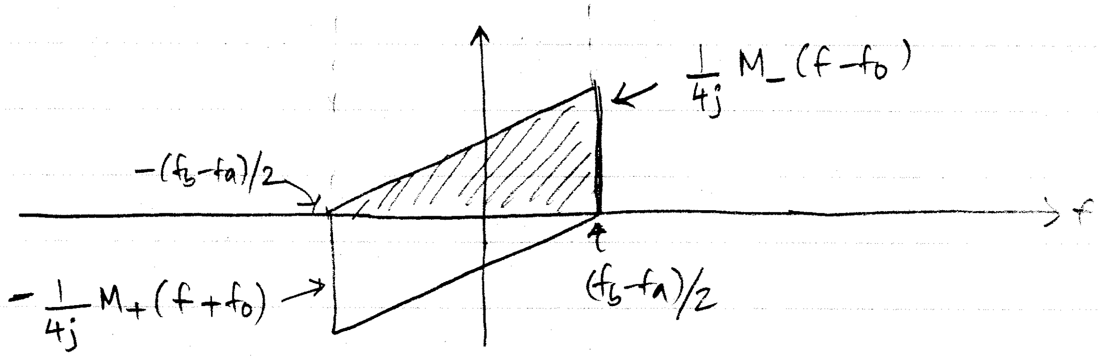
Output of first product mod. (Quadrature channel)



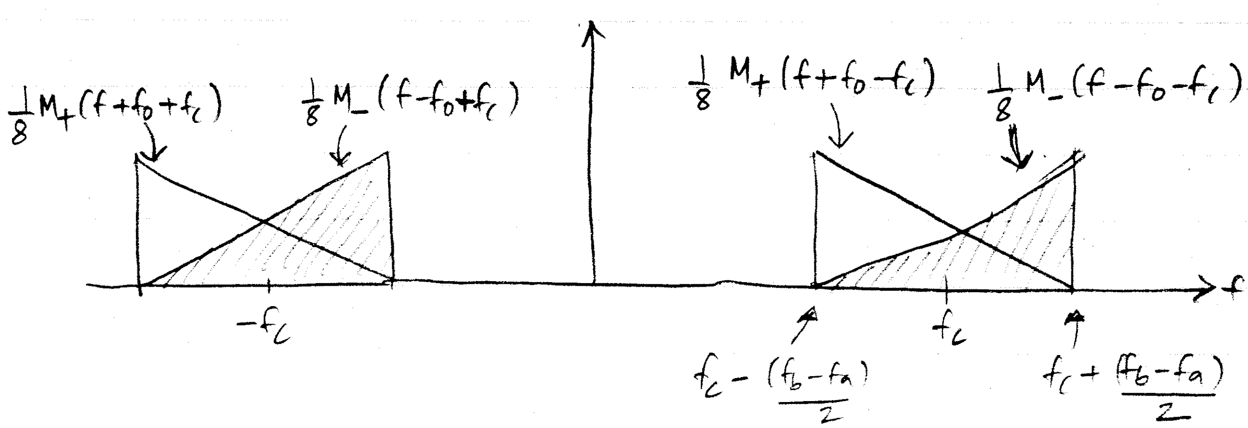
Output of LPF (in-phase)



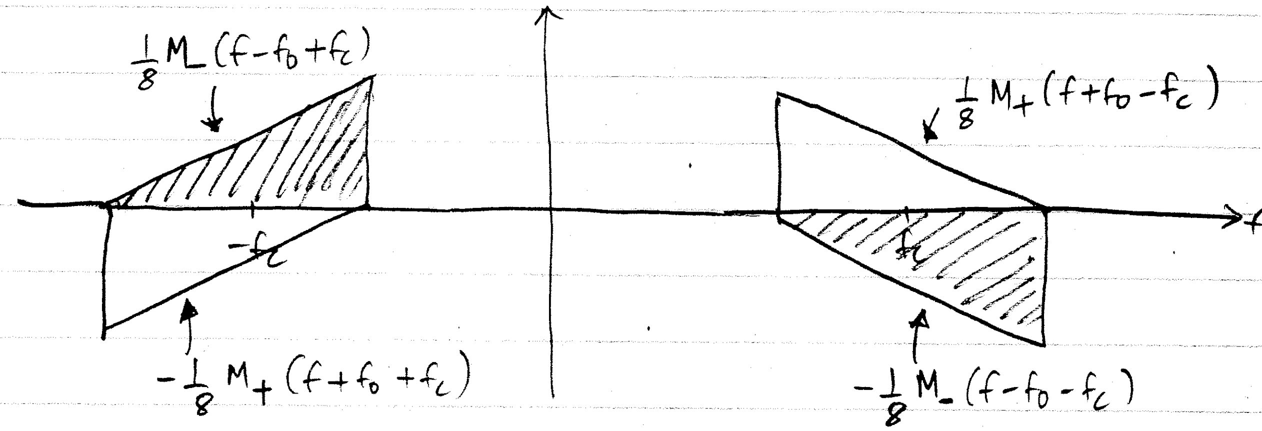
Output of LPF (quadrature)



Output of second product mod. (in-phase)



Output of second product mod. (Quadrature)



Adding the outputs of the two product modulators:

(a) For the lower SB, in-phase and quadrature components cancel each other.

(b) For the upper SB, in-phase and quadrature components are of same polarity, and by adding them upper SB is transmitted

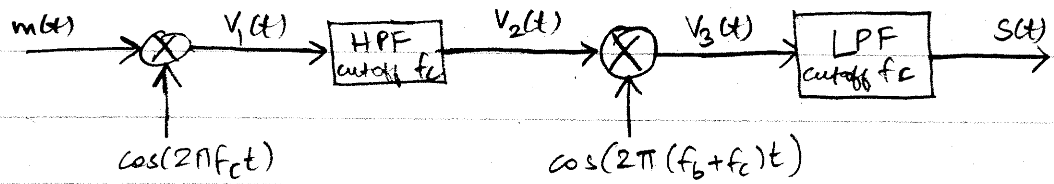
(c) How to transmit only lower SB?

A single sign change is enough!

For eg.: → Change to a -ve sign in the final summation block for quadrature channel.

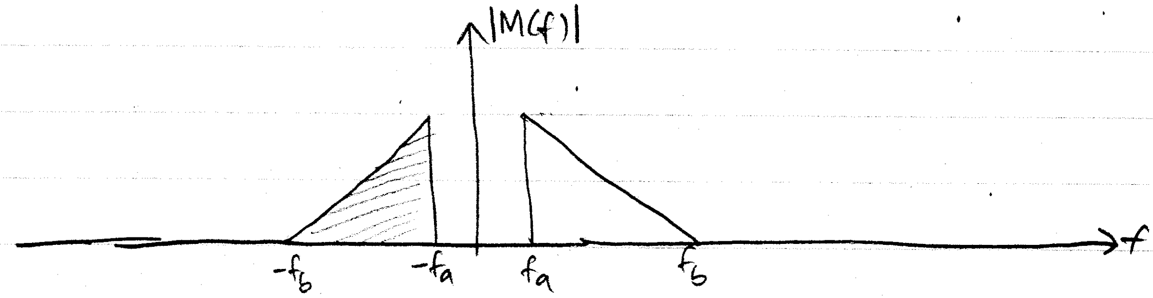
(or) → Do product modulation with $-\sin(2\pi f_c t)$ in quadrature channel

2.19



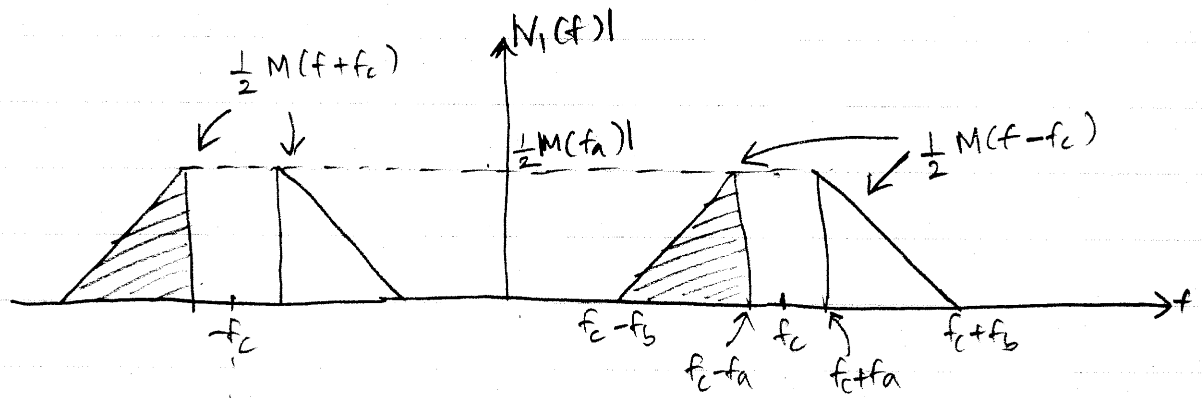
$(f_c > f_b)$

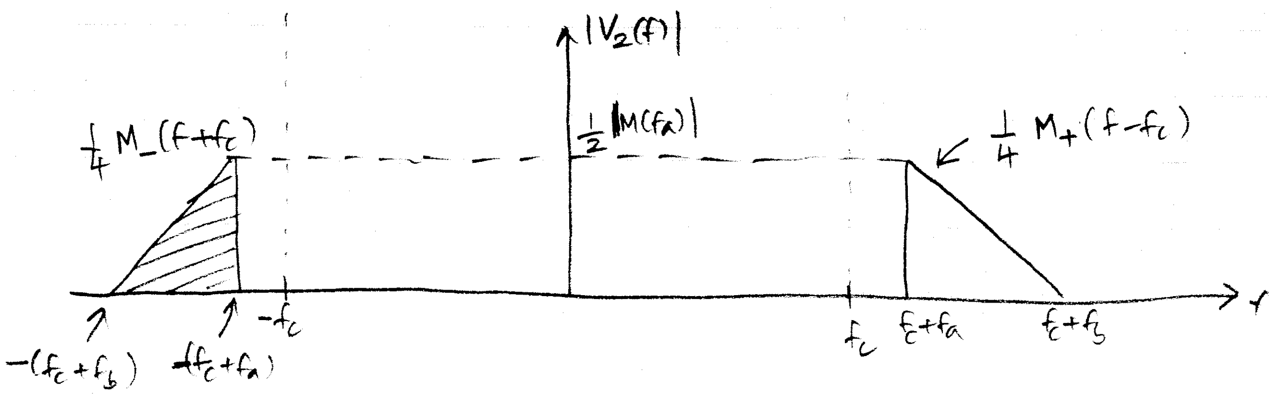
Suppose $m(t)$ has the following spectrum



$v_1(t) = \cos(2\pi f_c t)\, m(t)$

$V_1(f) = \frac{1}{2}\left[M(f-f_c) + M(f+f_c) \right]$



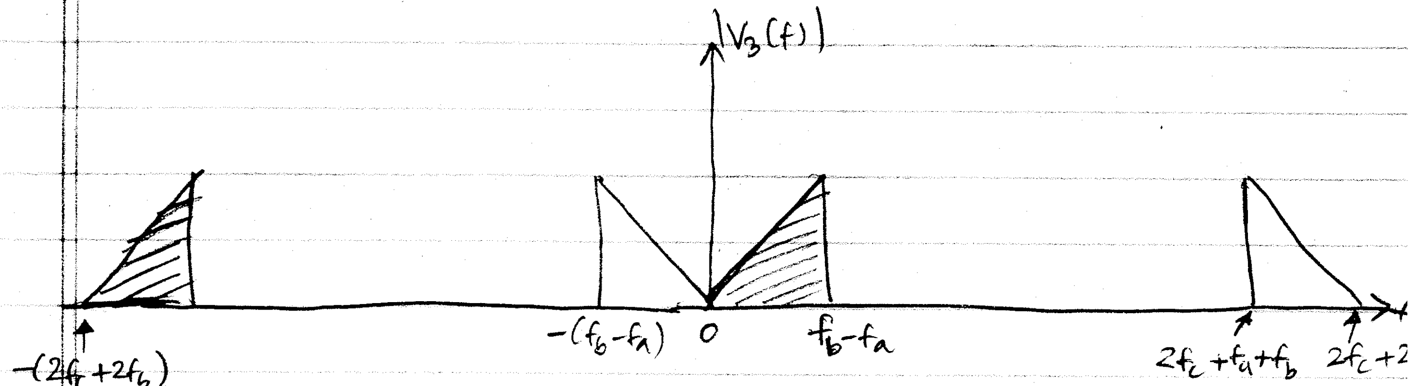


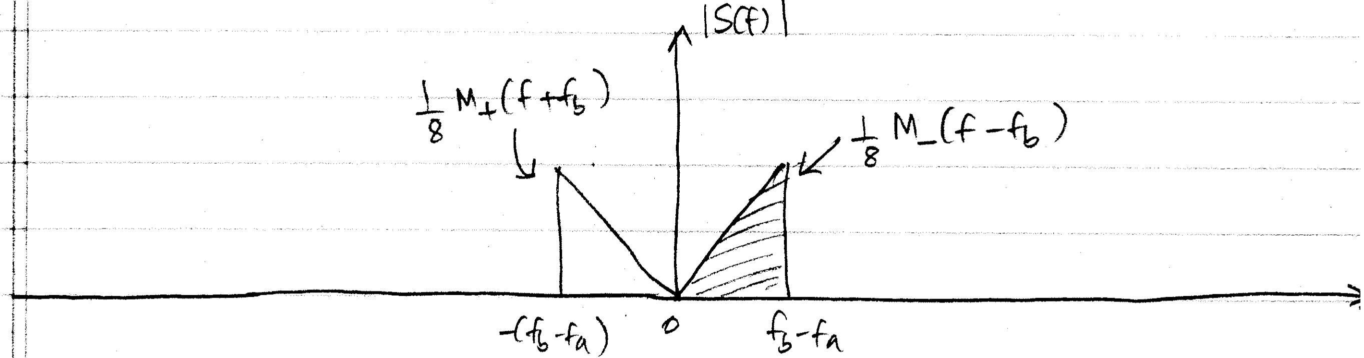
$$v_3(t) = v_2(t)\cos(2\pi(f_b+f_c)t)$$

$$V_3(f) = \frac{1}{2}\left[V_2(f-f_b-f_c) + V_2(f+f_b+f_c)\right]$$

$$= \frac{1}{8}\left[M_+(f-f_b-2f_c) + M_+(f+f_b)\right]$$

$$+ \frac{1}{8}\left[M_-(f-f_b) + M_-(f+f_b+2f_c)\right].$$





$$S(f) = \frac{1}{8}\left[M_+(f+f_b) + M_-(f-f_b)\right]$$

$$\therefore\ s(t) = \frac{1}{8}m_+(t)\exp(-j2\pi f_b t) + \frac{1}{8}m_-(t)\exp(j2\pi f_b t)$$

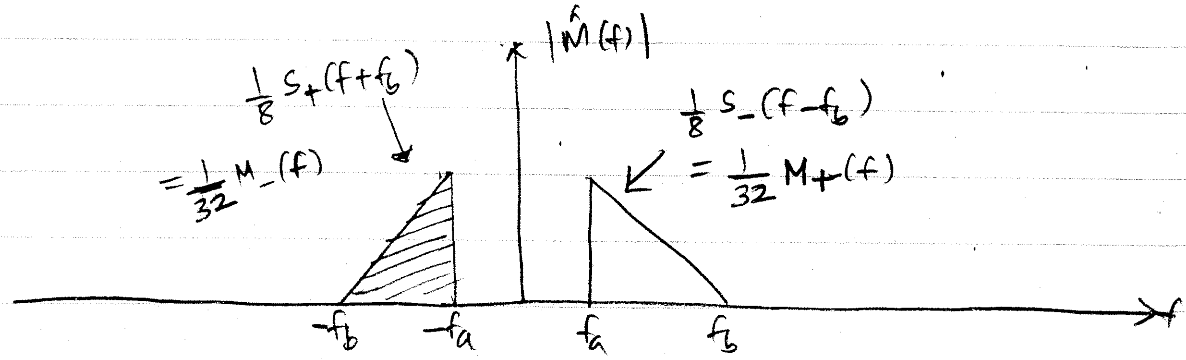
$$= \frac{1}{8}\left[m(t)+j\hat{m}(t)\right]\left[\cos(2\pi f_b t) - j\sin(2\pi f_b t)\right]$$

$$+ \frac{1}{8}\left[m(t)-j\hat{m}(t)\right]\left[\cos(2\pi f_b t) + j\sin(2\pi f_b t)\right]$$

$$= \frac{1}{4}m(t)\cos(2\pi f_b t) + \frac{1}{4}\hat{m}(t)\sin(2\pi f_b t)$$

(b) If $s(t)$ is used as input to the same circuit, then the output spectrum is

$$\hat{M}(f) = \frac{1}{8}\left[S_+(f+f_b) + S_-(f-f_b) \right]$$



$\therefore$ $m(t)$ is recovered !

2.21

(a) $s(t) = \frac{1}{2} a A_m A_c \cos[2\pi(f_c+f_m)t] + \frac{1}{2} A_m A_c (1-a) \cos[2\pi(f_c-f_m)t]$

$= \frac{1}{2} a \cancel{A_m A_c \cos(2\pi f_c t)} \cos(2\pi f_m t)$

$- \frac{1}{2} a A_m A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$

$+ \frac{1}{2} (1-\cancel{a}) A_m A_c \cos(2\pi f_c t) \cos(2\pi f_m t)$

$+ \frac{1}{2} (1-a) A_m A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$

$= \overbrace{\frac{1}{2} A_m A_c \cos(2\pi f_m t)}^{s_I(t)} \cos(2\pi f_c t)$

$+ \underbrace{\frac{1}{2}(1-2a) A_m A_c \sin(2\pi f_m t)}_{-s_Q(t)} \sin(2\pi f_c t)$

$\therefore$ quadrature component: $s_Q(t) = -\frac{1}{2}(1-2a) A_m A_c \sin(2\pi f_m t)$

(b) $s(t) + A_c \cos(2\pi f_c t)$

$= A_c \left[1 + \frac{1}{2} A_m \cos(2\pi f_m t)\right] \cos(2\pi f_c t)$

$+ \frac{1}{2} A_c A_m (1-2a) \sin(2\pi f_m t) \sin(2\pi f_c t)$

Envelope

$a(t) = \sqrt{A_c^2 \left[1 + \frac{1}{2} A_m \cos(2\pi f_m t)\right]^2 + A_c^2 \left[\frac{1}{2} A_m (1-2a) \sin(2\pi f_m t)\right]^2}$

(See Page 730 for a(t) formula)

$= A_c \left[1 + \frac{1}{2} A_m \cos(2\pi f_m t)\right] \sqrt{1 + \left(\frac{\frac{1}{2} A_m (1-2a) \sin(2\pi f_m t)}{1 + \frac{1}{2} A_m \cos(2\pi f_m t)}\right)^2}$

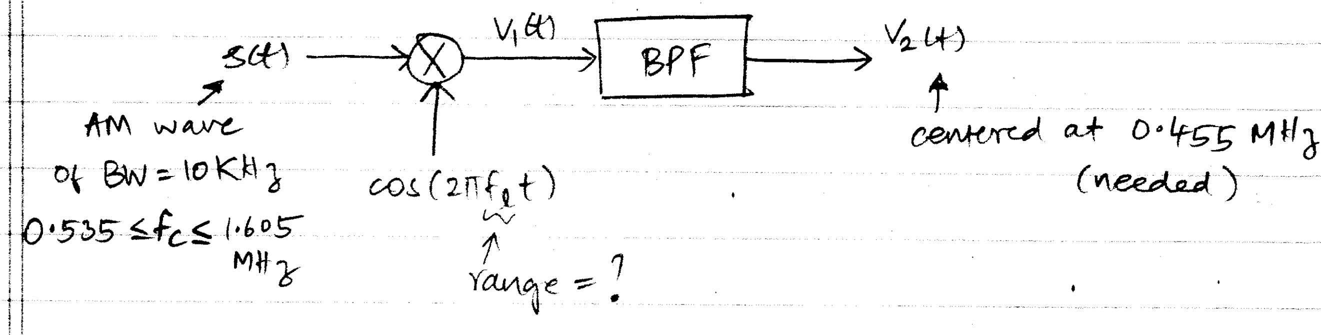
$$\therefore \quad a(t) = A_c\left(1 + \frac{1}{2} A_m \cos(2\pi f_m t)\right) d(t)$$

where

$$d(t) \triangleq \sqrt{1 + \left(\frac{\frac{1}{2} A_m (1-2a) \sin(2\pi f_m t)}{1 + \frac{1}{2} A_m \cos(2\pi f_m t)}\right)^2} \quad \leftarrow \underline{\text{distortion}}$$

(c) $d(t)$ is maximum when $\underline{\underline{a=0}}$ (which is also intuitive as 'a' is the attenuatio of the upper side freq.).

2.22 Superheterodyne receiver.



$\because$ the product modulator shifts freqs to $(f_c + f_l)$ and $(f_c - f_l)$, we need

$$f_c - f_l = 0.455 \text{ MHz}$$

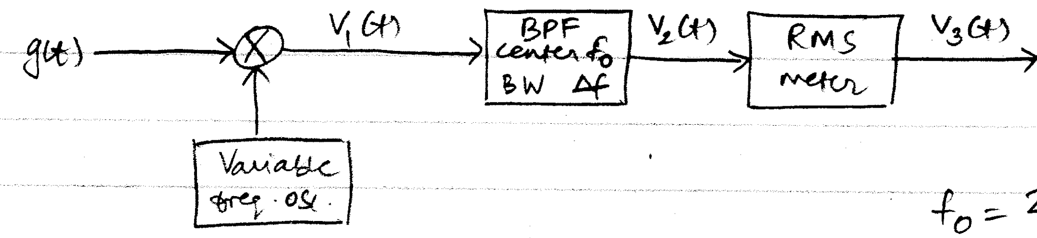
$$\begin{array}{r} 0.535 \\ 0.455 \\ \hline 0.080 \end{array}$$

i.e., $f_l = f_c - 0.455$ MHz.

$$\begin{array}{r} 1.605 \\ 0.455 \\ \hline 1.150 \end{array}$$

$\therefore 0.08 \le f_l \le 1.15$ MHz

2.23 Heterodyne spectrum analyzer.



amplitude : A

freq. range : f_0 to $f_0 + W$

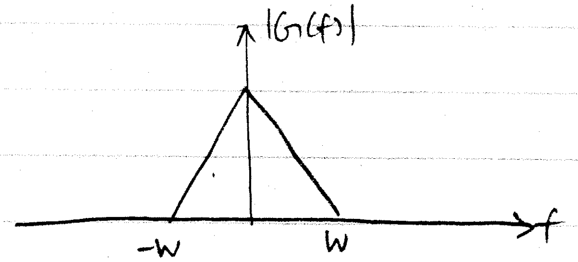
$f_0 = 2W$

$\Delta f << f_0$

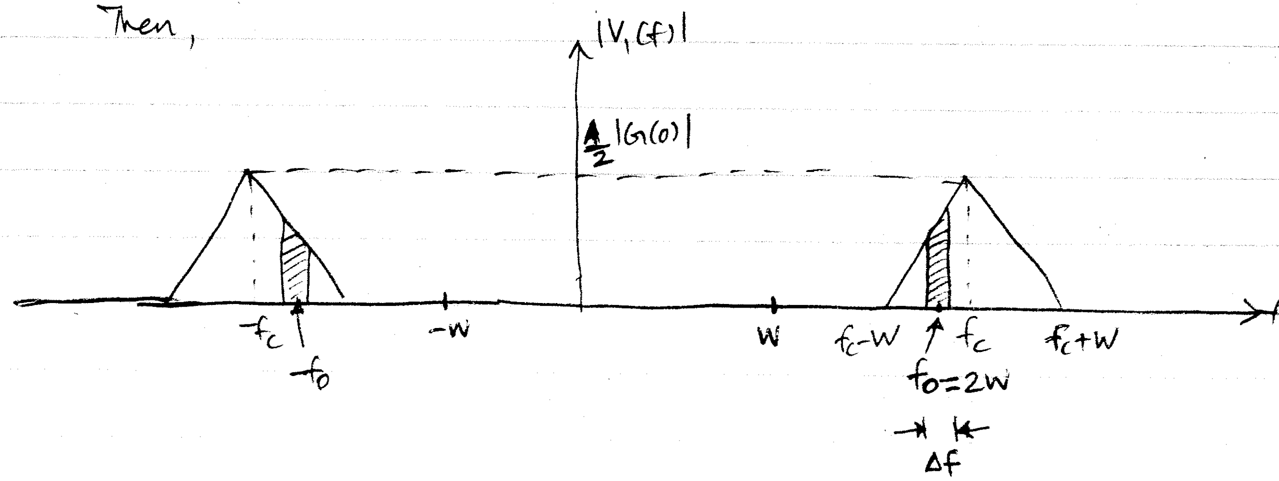
Let f_c denote the oscillator freq. $\therefore \quad f_0 \le f_c \le f_0 + W$.

$$V_1(t) = A\, g(t) \cos(2\pi f_c t)$$

Suppose $g(t)$ has the following spectrum.



Then,



$\because$ $\Delta f << f_0$,

$$V_2(f) \cong \frac{A}{2} G(f_c - f_0), \quad f_0 - \frac{\Delta f}{2} \le |f| \le f_0 + \frac{\Delta f}{2}$$

$$V_3(t) = \sqrt{\int_{-\infty}^{\infty} |v_2(t)|^2 dt}$$

$$= \sqrt{\int_{-\infty}^{\infty} |V_2(f)|^2 df} \qquad \text{(By Rayleigh's energy theorem)}$$

$$= \left[\frac{1}{4} A^2 |G(f_c - f_0)|^2 \cdot 2\Delta f \right]^{1/2}$$

$$= \frac{A}{\sqrt{2}} |G(f_c - f_0)| \cdot \sqrt{\Delta f}$$