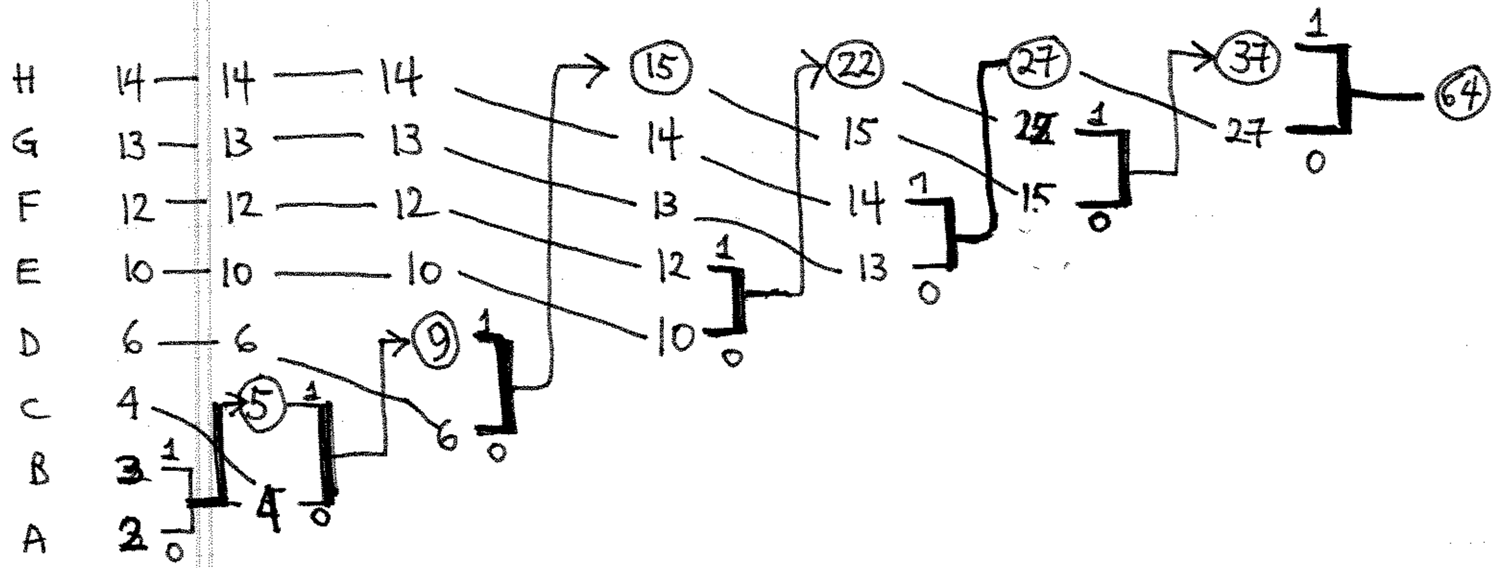
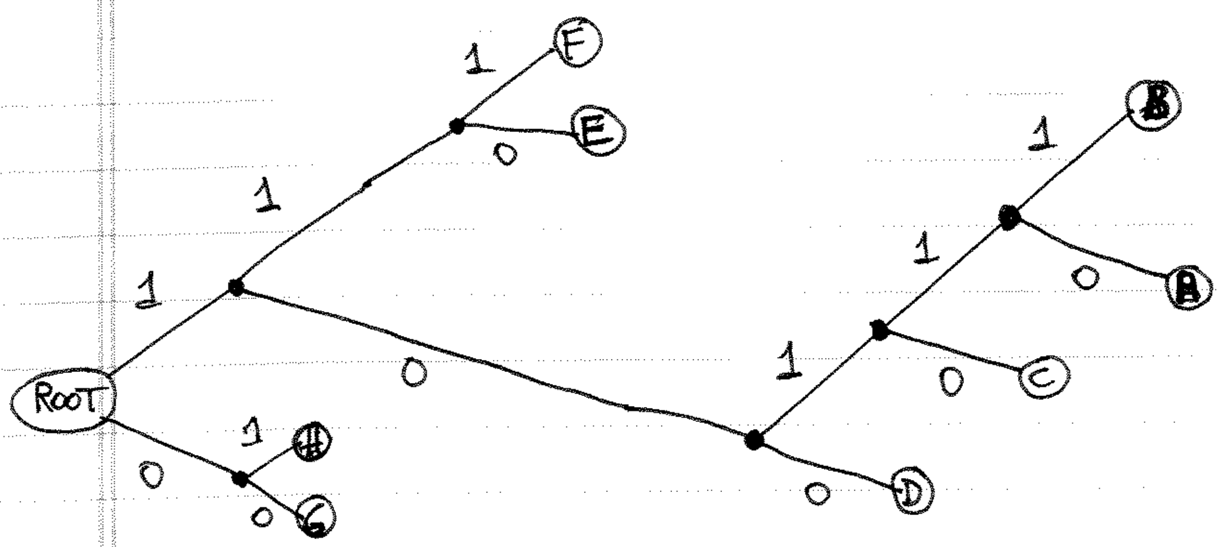
ENEE 420
Test #1

#1 a)



The Huffman is (essentially) unique - Reading in reverse order



b/

A	10110
B	10111
C	1010
D	100
E	110
F	111
G	00
H	01

(Essentially) Unique Huffman Code

c/ The smallest average code length is the average code length for the Huffman code in 1b/, namely

$$L_{min} = \frac{1}{64}\left(5\times(2+3) + 4\times 4 + 3\times(6+10+12) + 2\times(13+14)\right)$$

$$= \frac{1}{64}\left(5\times 5 + 4\times 4 + 3\times 28 + 2\times 27\right)$$

$$= \frac{1}{64}\left(25 + 16 + 84 + 54\right)$$

$$= \frac{1}{64}\left(25 + 100 + 54\right) = \frac{179}{64}$$

We have $H(p) \neq \frac{179}{64}$ because $p_i \neq 2^{-n_i}$ for some integer n_i

e.g. $p_c = 3\times 2^{-6}$!

#2

a/ Phrases

N	(binary)		Bit stream generated by LZ alg.
0	00000	λ	
1	00001	0	00000 0
2	00010	00	00001 0
3	00011	1	00000 1
4	00100	001	00010 1
5	00101	10	00011 0
6	00110	000	00010 0
7	00111	11	00011 1
8	01 000	100	00101 0
9	01 001	111	00111 1
10	01 010	1000	01 000 0
11	01 011	110	00 111 0
12	01 100	0011	00100 1
13	01 101	01	00001 1
14	01 110	10001	01010 1
15	01 111	1101	01011 1
16	10 000	101	00101 1
17	10 001	1100	01011 0

c/ We parse in blocks of size 6 (5+1):

			N innovation bit		
1	00000	0	0 + 0	0	
2	00001	0	1 + 0	00	
3	00000	1	0 + 1	1	
4	00010	1	2 + 1	001	
5	00011	0	3 + 0	10	
6	00010	0	2 + 0	000	
7	00011	1	3 + 1	11	
8	00101	0	5 + 0	100	
9	00111	1	7 + 1	111	
10	01000	0	8 + 0	1000	
11	00111	0	7 + 0	110	
12	01000	1	8 + 1	1001	← ERROR
13	00001	1	1 + 1	01	
14	01010	1	10 + 1	10001	
15	01011	1	11 + 1	1101	
16	00100	1	4 + 1	0011	← ERROR
17	01011	0	11 + 0	1100	

Decoder will not know that errors are present since the parsing does not yield any contradiction, i.e., point to entries in the dictionary not yet generated. Also, none of the codewords is duplicated!

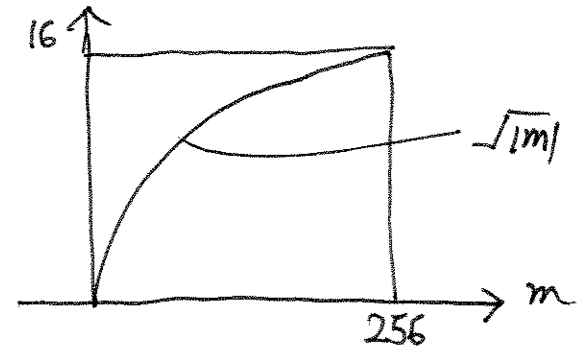
d We parse again

1	00000	0	0 + 0
2	00001	0	1 + 0
3	00000	1	0 + 1
4	00010	1	2 + 1
5	00011	0	3 + 0
6	00010	0	2 + 0
7	00011	1	3 + 1
8	00101	0	5 + 0
9	00111	1	7 + 1
10	11000	0	24 + 0
11	00111	0	7 + 0
12	01000	1	8 + 1
13	00001	1	1 + 1
14	01010	1	10 + 1
15	01011	1	11 + 1
16	00101	1	5 + 1
17	01011	0	11 + 0

← ERROR – Will be detected since Entry N=24 has not been built yet!

#3

a/ Here $R=5$ so that $L = 2^R = 2^5 = 32$. The uniform quantizer has 32 levels and operates on the interval $(-16, 16)$ (since $\sqrt{256} = 16$).



$\therefore$ The cells $C_1, \cdots, C_L$ are simply

$$(-16,-15), \cdots, (-1,0), (0,1), \cdots, (15,16)$$

with associated levels

$$-15.5, -14.5, \cdots, -0.5, 0.5, \cdots, 14.5, 15.5$$

b/ The cells $\tilde{C}_1, \cdots, \tilde{C}_L$ are simply

$$(-256,-225), \cdots, (-9,-4)(-4,-1)(-1,0)(0,1)(1,4)(4,9), \cdots, (225,256)$$

with associated values $\tilde{V}_1, \cdots, \tilde{V}_L$

$$-15.5, \cdots, -0.5, 0.5, \cdots, 14.5, 15.5$$

So the quantized version of $m = 60$ is simply 7.5 since $7^2 < 60 < 8^2$.

c/ If $v = \Phi(m)$, $|m| \leq 256$, then the expansion characteristic is

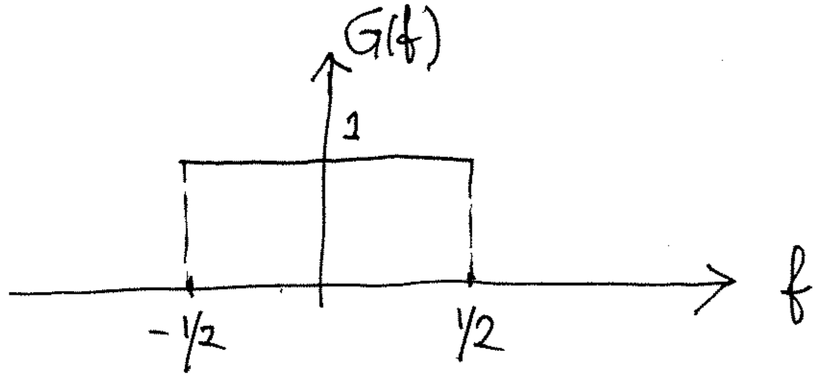
$$m = \Phi^{-1}(v) = v^2 \operatorname{sgn}(v) \;, \quad v \in \mathbb{R}.$$

d/ The use of Ψ, instead of Φ, means that the interval $(-256, 256)$ is now mapped into itself (instead of $(-16, 16)$). This yields a better representation of signals in PCM-like systems since "magnitude is preserved".

#4

a) Since

$$\frac{\sin \pi t}{\pi t} \longleftrightarrow \text{rectangular pulse} \begin{cases} 1 & \text{if } |t| \le \frac{1}{2} \\ 0 & \text{if } |t| > \frac{1}{2} \end{cases}$$



The signal g is band-limited with cutoff $W = \frac{1}{2}$ so that $f^* = 2 \cdot \frac{1}{2} = 1$.

b) Here

$$g_{PAM}(t) = (g_\delta * p)(t), \quad t \in \mathbb{R}$$

so that

$$G_{PAM}(f) = G_\delta(f)\, P(f)$$

$$= f_s \sum_k G(f - kf_s) \cdot P(f), \quad f \in \mathbb{R}$$

with $f_s = \frac{1}{T_s}$ and

$$P(f) = \int_0^\tau e^{-j2\pi f t}\, dt$$

$$= \frac{e^{-j2\pi f \tau} - 1}{-j2\pi f}$$

$$= \frac{e^{j\pi f \tau} - e^{-j\pi f \tau}}{2j\pi f}\, e^{-j\pi f \tau}$$

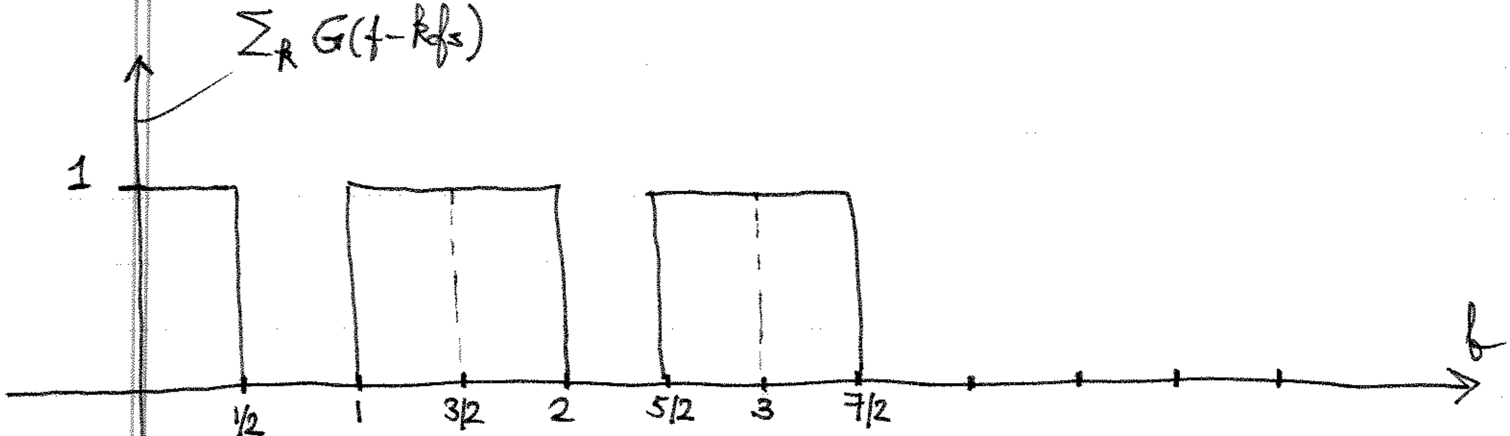
$$= \frac{\sin(\pi f \tau)}{\pi f}\, e^{-j\pi f \tau}, \quad f \in \mathbb{R}.$$

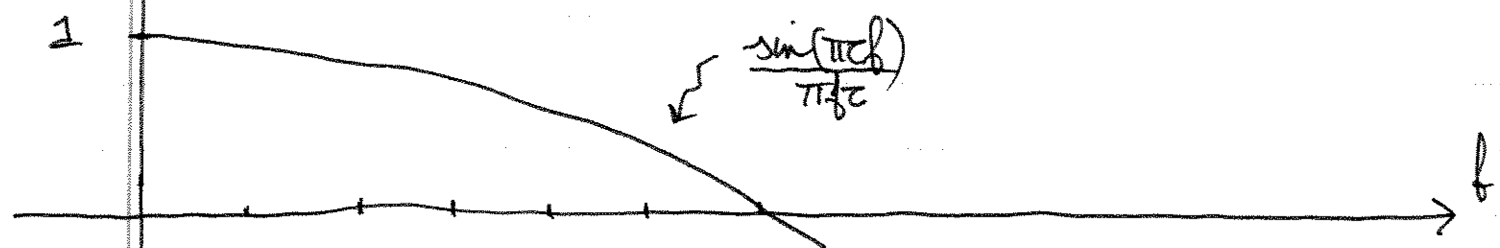
$$\therefore \quad G_{PAM}(f) = f_s \left[\sum_k G(f - kf_s)\right] \frac{\sin(\pi f \tau)}{\pi f}\, e^{-j\pi f \tau}, \quad f \in \mathbb{R}$$

$$\therefore \quad |G_{PAM}(f)| = \tau f_s \left[\sum_k G(f - kf_s) \right] \left| \frac{\sin(\pi f \tau)}{\pi f \tau} \right| , \quad f \in \mathbb{R} .$$

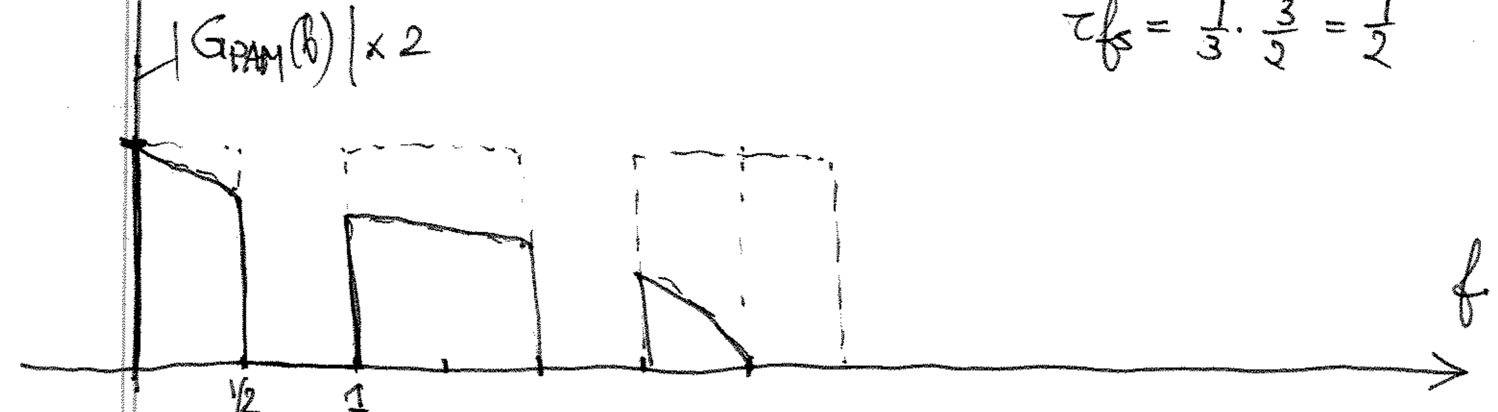
Here $T_s = \frac{2}{3}$ so $f_s = \frac{3}{2}$

$\tau = \frac{1}{3}$ so $\sin(\pi f \tau) = 0$ @ $\hat{f} = \frac{1}{\tau} = 3$

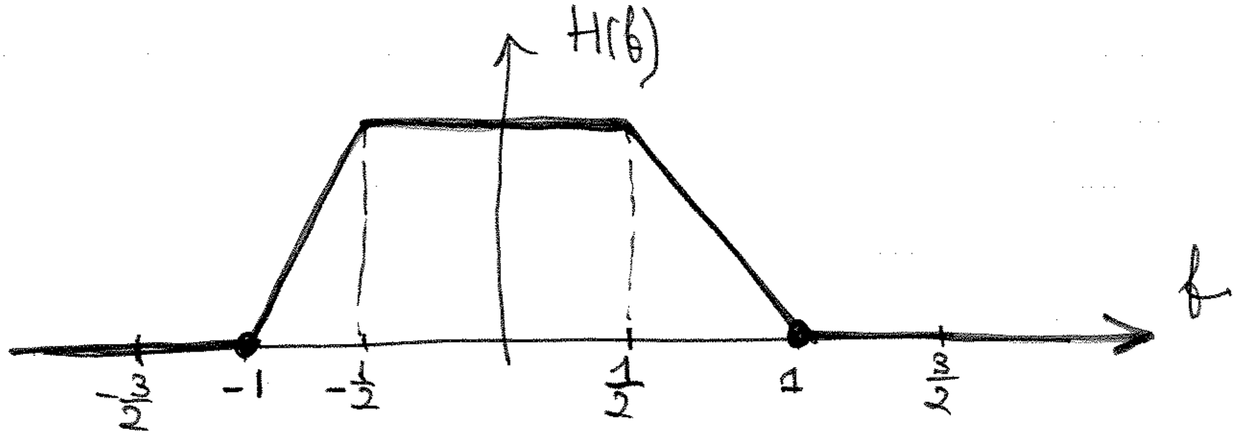




$\tau f_s = \frac{1}{3} \cdot \frac{3}{2} = \frac{1}{2}$



c/ Use the LP filter $H(f)$ given by



The maximum amplitude distortion occurs at $f=\frac{1}{2}$ and is equal to

$$\tau f_s \cdot \left| \frac{\sin(\pi f \tau)}{\pi f \tau} \right|$$

$$= \frac{1}{2} \frac{\sin(\pi/6)}{\pi/6}$$

$$= 3 \frac{\sin(\pi/6)}{\pi}$$

$$= \frac{3}{\pi} \cdot \frac{1}{2} = \frac{3}{2\pi} \quad \#.$$