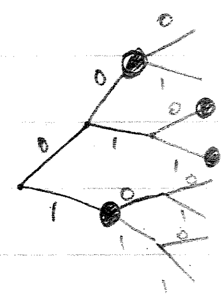
9/17/07
Recitation

* $\{1, 2, 3, 3\}$ ← given lengths

construct prefix code.



Given $\{l(x), x \in \mathcal{X}\}$

① Take some $x \in \mathcal{X}$

② Label the first node lexicographically of depth $l(x)$ as $C(x)$

③ Delete "descendants" of $C(x)$

④ Repeat for other x

9.7

$\mathcal{S} = \{s_0, s_1, s_2\}$

$P = \{0.7, 0.15, 0.15\}$

(a) $H_2(S) = -0.7 \log_2 0.7 - 0.15 \log_2 0.15 - 0.15 \log_2 0.15$

$= 1.079$ bits

(b) S^2 = second order extension

$\mathcal{S} \times \mathcal{S}$

$P(s_i, s_j) = p(s_i)\, p(s_j)$

(s_i, s_j)	(s_0, s_0)	(s_0, s_1)	(s_0, s_2)	(s_1, s_0)	(s_1, s_1)	(s_1, s_2)
$p(s_i, s_j)$	0.49	0.105	0.105	0.105	0.0225	0.0225

(s_2, s_0)	(s_2, s_1)	(s_2, s_2)
0.105	0.0225	0.0225

$$H(S^2) = -\sum_{i,j=0}^{2} p(s_i, s_j) \log_2 p(s_i, s_j)$$

$= 2 \times 1.079 = 2.158$ bits

9.6

$$\mathcal{S} = \{s_0, \ldots, s_{K-1}\}$$

$$\mathcal{P} = \{p_0, \ldots p_{K-1}\}$$

$$\mathcal{S}^n = \mathcal{S} \times \cdots \times \mathcal{S}$$

$$= \{\sigma_0, \ldots \sigma_{M-1}\} \quad , \; M = K^n$$

$H(S^n) = n\,H(S)$

(a) $$\sum_{i=0}^{M-1} P(\sigma_i) = 1$$

$$P(\sigma_i) = P(s_{i_1} s_{i_2} \cdots s_{i_n})$$

$$= P(s_{i_1}) \cdot P(s_{i_2}) \cdots P(s_{i_n})$$

$$= \sum_{i_1=0}^{K-1} \sum_{i_2=0}^{K-1} \cdots \sum_{i_n=0}^{K-1} P(s_{i_1}) \cdots P(s_{i_n})$$

$$= \sum_{i_1=0}^{K-1} P(s_{i_1}) \cdot \sum_{i_2=0}^{K-1} P(s_{i_2}) \cdots \sum_{i_n=0}^{K-1} P(s_{i_n})$$

$$= 1$$

(b) For $k = 1, 2, \ldots, n$

$$\sum_{i=0}^{M-1} P(\sigma_i) \log_2\left(\frac{1}{P(s_{i_k})}\right) = H_2(S)$$

$$= \sum_{i_1=0}^{K-1} \cdots \sum_{i_n=0}^{K-1} P(s_{i_1}) \cdots P(s_{i_n}) \cdot \log_2\left(\frac{1}{P(s_{i_k})}\right)$$

$$= \sum_{i_1=0}^{K-1} P(s_{i_1}) \cdots \sum_{i_k=0}^{K-1} P(s_{i_k}) \log_2 \frac{1}{P(s_{i_k})} \cdots \sum_{i_n=0}^{n} P(s_{i_n})$$

$$= H_2(S)$$

(c) $$H(S^n) = \sum_{i=0}^{M-1} P(\sigma_i) \log_2 \frac{1}{P(\sigma_i)}$$

$$= \sum_{i_1=0}^{K-1} \cdots \sum_{i_n=0}^{K-1} P(s_{i_1}) \cdots P(s_{i_n}) \log_2 \frac{1}{P(s_{i_1}) \cdots P(s_{i_n})}$$

$$= \sum_{i_1=0}^{K-1} \cdots \sum_{i_n=0}^{K-1} P(s_{i_1}) \cdots P(s_{i_n}) \left[\log_2 \frac{1}{P(s_{i_1})} + \cdots + \log \frac{1}{P(s_{i_n})} \right]$$

$$= \sum_{i_1=0}^{K-1} \cdots \sum_{i_n=0}^{K-1} P(s_{i_1}) \cdots P(s_{i_n}) \log_2 \frac{1}{P(s_{i_1})}$$

$$+ \cdots + \sum_{i_1=0}^{K-1} \cdots \sum_{i_n=0}^{K-1} P(s_{i_1}) \cdots P(s_{i_n}) \log_2 \frac{1}{P(s_{i_n})}$$

$$= n \underline{\underline{H_2(S)}}$$