Recitation - 9/24/07

9.13

$\mathcal{S} = \{s_0, s_1, s_2\}$

$\mathcal{P} = \{0.7, 0.15, 0.15\}$

$S = (\mathcal{S}, \mathcal{P})$

$H_2(\mathcal{P}) = -0.7 \log_2 0.7 - 0.15 \log_2 0.15 - 0.15 \log_2 0.15$

$= 1.079$ bits

(a)

| | | | | |
|---|---|---|---|---|
| $s_0$ | 0.7 | $\rightarrow$ 0.7 | 0 | |
| $s_1$ | 0.15 | 0 | $\rightarrow$ 0.3 | 1 |
| $s_2$ | 0.15 | 1 | | |

Huffman code

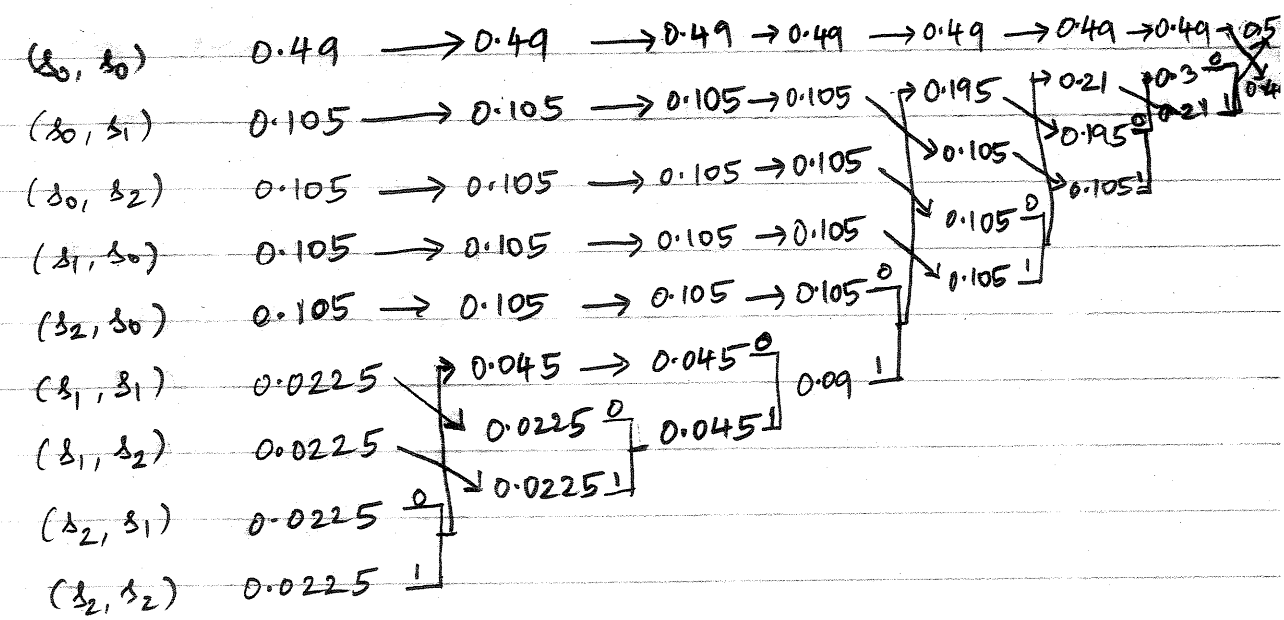
$s_0$: 0

$s_1$: 1 0

$s_2$: 1 1

$L(C, \mathcal{P}) = 0.7 \times 1 + 0.15 \times 2 + 0.15 \times 2 = 1.3$ bits/symbol

(b) Extend source to order two.



| | | |
|---|---|---|
| $(s_0, s_0)$ | 1 | |
| $(s_0, s_1)$ | 001 | |
| $(s_0, s_2)$ | 010 | |
| $(s_1, s_0)$ | 011 | |
| $(s_2, s_0)$ | 0000 | |
| $(s_1, s_1)$ | 000110 | |
| $(s_1, s_2)$ | 000111 | |
| $(s_2, s_1)$ | 000100 | |
| $(s_2, s_2)$ | 000101 | |

$$L(C_2, P_2) = 0.49 \times 1 + 0.105 \times 3 \times 3 + 0.105 \times 4 + 0.0225 \times 6 \times 4$$

$= 2.395$ bits / 2 symbols.

i.e. $1.1975$ bits/symbol.

(c) $H_2(P) = 1.079$ bits/symbol

$L(C, P) = 1.3$ bits/symbol

$L(C_2, P_2) = 1.1975$ bits/symbol

$\therefore$ by extending the source, the average code-word length/symbol has become closer to the entropy of the original source.

In the class, we saw that

$$H_2(P) \leq L(C^*, P) \leq H_2(P) + 1$$

and for the extended source

$$H_2(P_n) \leq L(C_n^*, P_n) \leq H_2(P_n) + 1$$

i.e., $$H_2(P) \leq \frac{L(C_n^*, P_n)}{n} \leq H_2(P) + \frac{1}{n}.$$

This is indeed true for the example above:

$$1.079 \leq L(C, P) = 1.3 \leq 2.079$$

$$1.079 \leq L(C_2, P) = 1.1975 \leq 1.579$$

* Also, covered solutions to HW1.