Recitation : 10/15/07

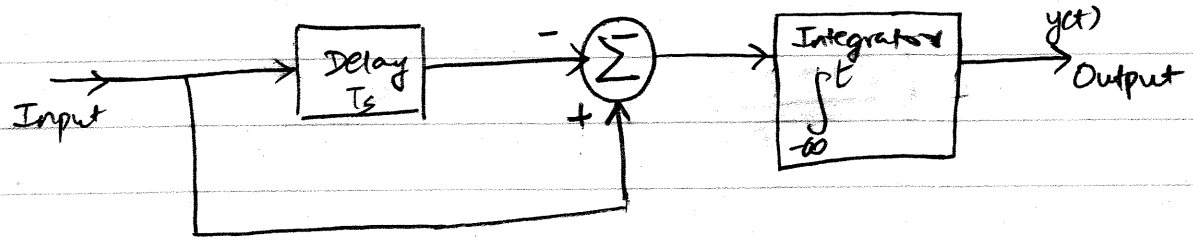
P A bandlimited signal $x(t)$ is sampled faster than the corresponding Nyquist rate, with sampling period T_s sec. The sampling is ideal, i.e, performed using the sampling function

$$\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) .$$

The resulting sampled signal is

$$x_s(t) = x(t)\,\delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t - nT_s) .$$

At the receiver, the following "zero-order circuit" is sometimes used to reconstruct a signal from its sampled version $x_s(t)$.



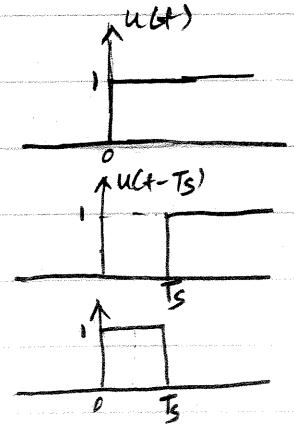
(a) Determine the impulse response of this circuit.

i.e. we need to determine the output when $\delta(.)$ is applied as input.

$$y(t) = \int_{-\infty}^{t} \left(\delta(\tau) - \delta(\tau - T_s) \right) d\tau$$

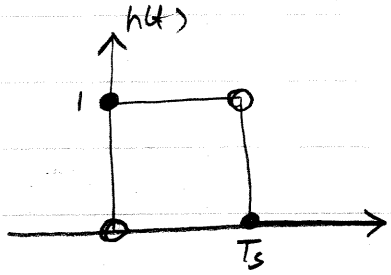
$$= u(t) - u(t - T_s)$$

$$= \text{rect}\left(\frac{t - T_s/2}{T_s} \right)$$



∴ impulse response

$$h(t) = \text{rect}\left(\frac{t - T_s/2}{T_s}\right)$$



(b) Write a time-domain expression for the output of the circuit when the sampled signal $x_s(t)$ is its input.

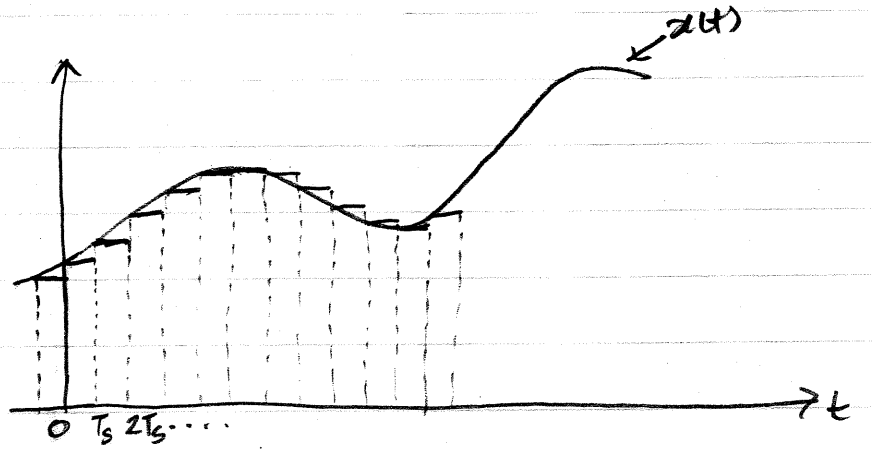
$$y(t) = x_s(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x_s(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(\tau - nT_s)\, \text{rect}\left(\frac{t-\tau-T_s/2}{T_s}\right) d\tau$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\, \text{rect}\left(\frac{t - T_s/2 - nT_s}{T_s}\right)$$

"Staircase approximation" of $x(t)$:



P

A signal of bandwidth 3KHz is sampled at 1.5 times its Nyquist rate, quantized by 200-level quantizer, and encoded into a binary PCM wave. The bit rate of the PCM output is ?

$f_s = \frac{3}{2} \times 6\text{KHz} = 9\text{ KHz}$ i.e. 9000 samples/sec.

bits per sample $= \lceil \log_2 200 \rceil = 8$ bits/sample.

Bit rate = 72 Kbps.

P

Suppose that 20 voice channels, each carrying a signal of bandwidth 2 KHz sampled at 2.5 times the Nyquist rate, are multiplexed in a TDMA frame. Each sample is quantized into one of 128 levels. The overall bit rate of this carrier system is ?

$f_s = 2.5 \times 4 = 10\text{ KHz} \Rightarrow T_s = 0.1\text{ ms}$

samples/sec in multiplexed frame = $20 \times 10\text{K} = 200$ K samples/sec

bits/sample $= \lceil \log_2 128 \rceil = 7$ bits/sample

Overall bit rate $= 7 \times 200\text{K} = 1.4$ Mbps