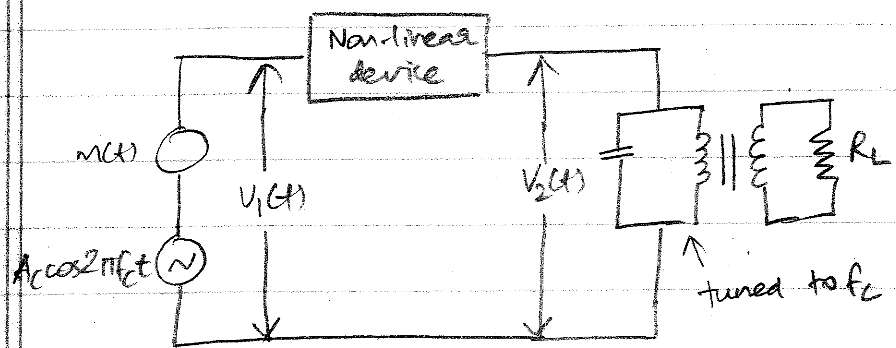
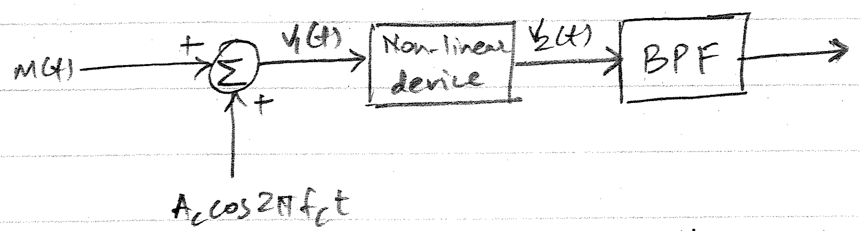
Recitation: 11/05/07

2.2 Square-law modulator



$V_1(t) = m(t) + A_c \cos 2\pi f_c t$

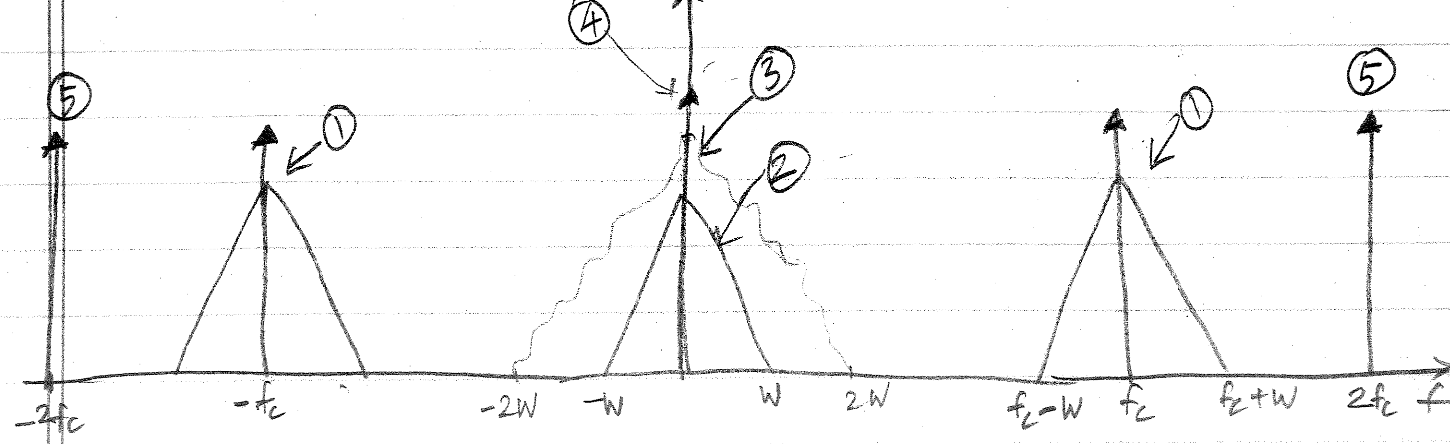
$V_2(t) = a_1 V_1(t) + a_2 V_1^2(t)$



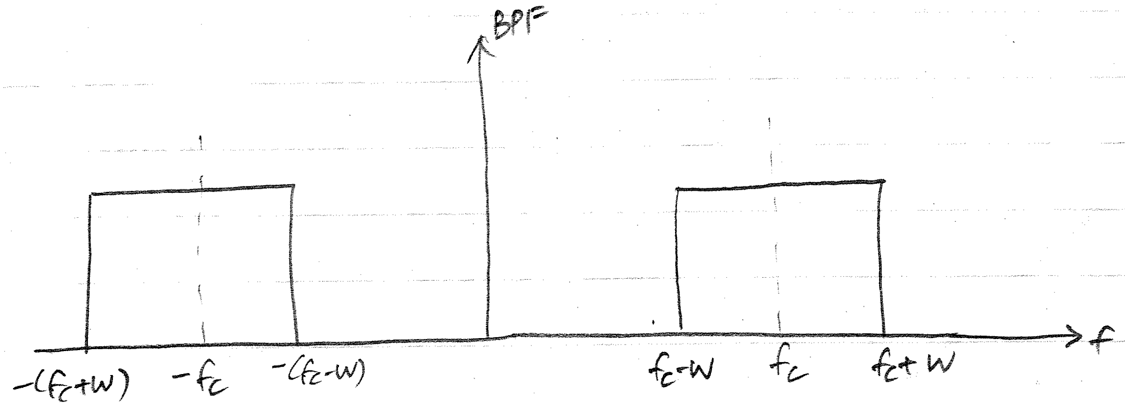
(a) $$V_2(t) = a_1\left[m(t) + A_c\cos 2\pi f_c t\right] + a_2\left[m^2(t) + A_c^2\cos^2 2\pi f_c t + 2m(t)A_c\cos 2\pi f_c t\right]$$

$$= \underbrace{a_1 A_c\left[1 + \frac{2a_2}{a_1}m(t)\right]\cos 2\pi f_c t}_{①} \quad \leftarrow \text{AM signal}$$

$$+ \underbrace{a_1 m(t)}_{②} + \underbrace{a_2 m^2(t)}_{③} + \underbrace{\frac{a_2 A_c^2}{2}}_{④} + \frac{a_2 A_c^2}{2}\underbrace{\cos(4\pi f_c t)}_{⑤}$$



(b)



Need: ① $f_c - W > 2W \iff f_c > 3W$

② $f_c + W < 2f_c \iff f_c > W$

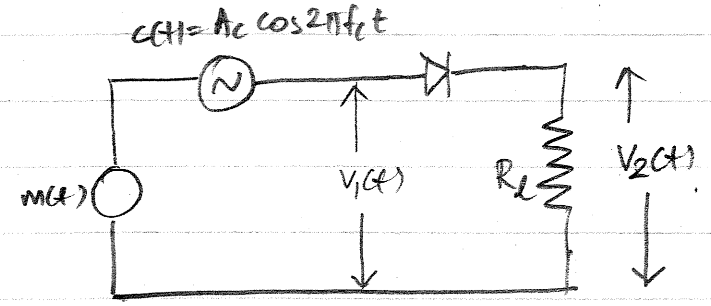
$\Rightarrow \underline{\underline{f_c > 3W}}$

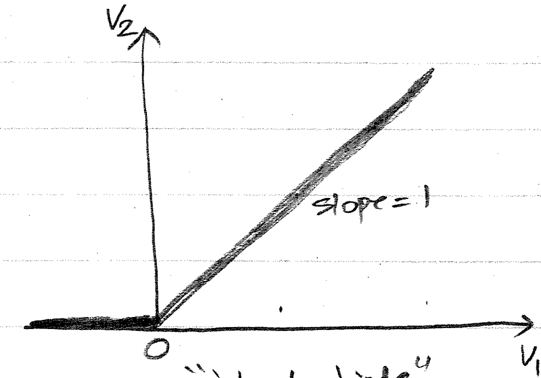
(c) Output of BPF $= a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t) \right] \cos 2\pi f_c t$

$$\Rightarrow k_a = \frac{2a_2}{a_1}$$

2.3

Switching Modulator





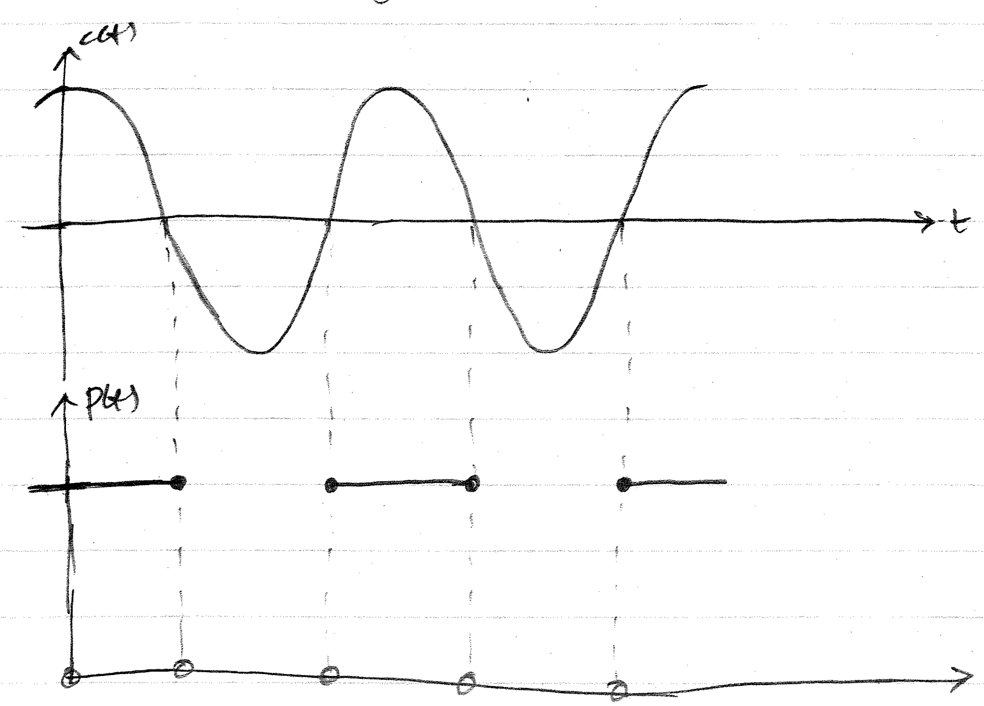
"ideal diode"

Assume: $A_c >> |m(t)|$

So $\quad V_1(t) \geq 0 \overset{\sim}{\Leftrightarrow} c(t) \geq 0$

$\quad\quad V_1(t) < 0 \overset{\sim}{\Leftrightarrow} c(t) < 0$

i.e. $\quad V_2(t) = \begin{cases} V_1(t) & \text{if } c(t) \geq 0 \\ 0 & \text{otherwise} \end{cases}$ $\quad$ (approx.)



$V_2(t) = V_1(t)\, p(t)$

$\quad\quad = \left(m(t) + A_c \cos 2\pi f_c t\right) p(t)$

Fourier series expansion:

$$p(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi f_c t(2n-1)\right]$$

odd multiples of f_c

(a) $v_2(t) \cong \left(m(t) + A_c \cos 2\pi f_c t\right) p(t)$

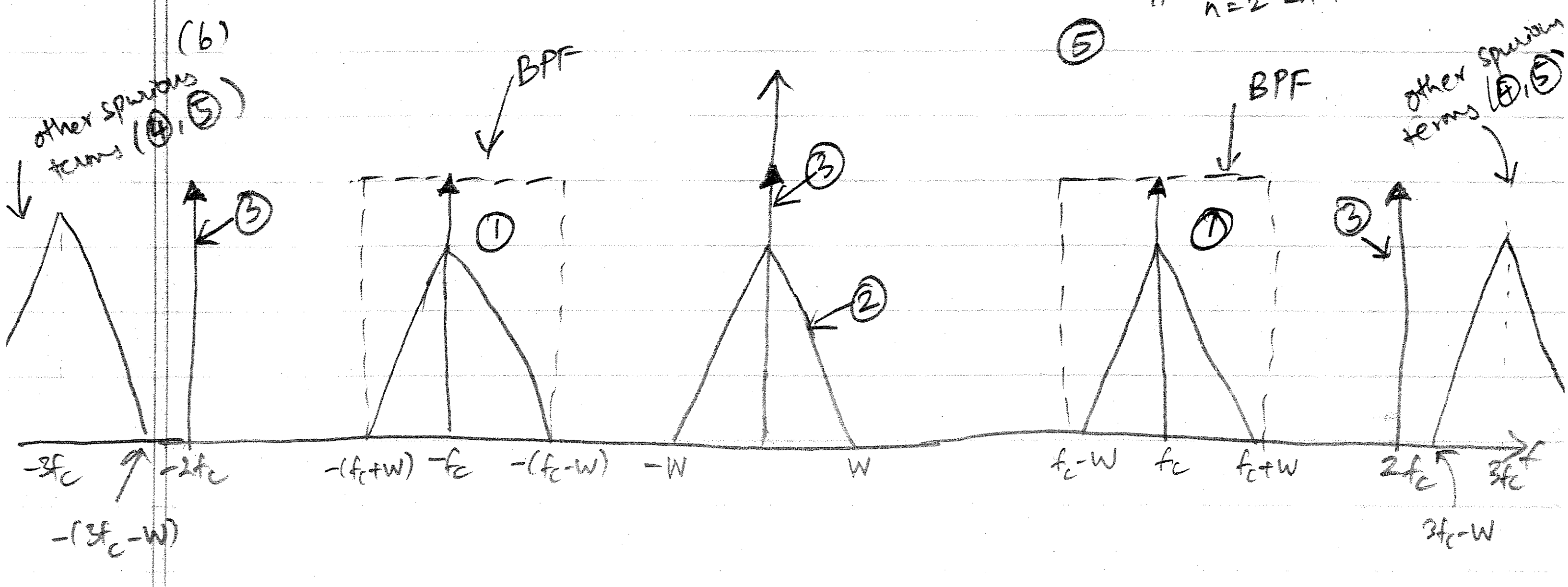
$$= \left(m(t) + A_c \cos 2\pi f_c t\right) \times$$
$$\left[\frac{1}{2} + \frac{2}{\pi}\cos 2\pi f_c t + \frac{2}{\pi}\sum_{n=2}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos\left[2\pi f_c t(2n-1)\right]\right]$$

$$= \frac{m(t)}{2} + \frac{2}{\pi} m(t)\cos 2\pi f_c t + \frac{A_c}{2}\cos 2\pi f_c t + \frac{2A_c}{\pi}\cos^2 2\pi f_c t$$
$$+ \left(m(t) + A_c \cos 2\pi f_c t\right) \times \frac{2}{\pi}\sum_{n=2}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos\left(2\pi f_c t(2n-1)\right)$$

$$= \underbrace{\frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right]\cos 2\pi f_c t}_{①}$$ ← AM signal

$$+ \underbrace{\frac{m(t)}{2}}_{②} + \underbrace{\frac{2A_c}{\pi}\cos^2 2\pi f_c t}_{③} + \underbrace{m(t)\cdot\frac{2}{\pi}\sum_{n=2}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos(2\pi f_c t(2n-1))}_{④}$$
$$+ \underbrace{(A_c \cos 2\pi f_c t)\cdot\frac{2}{\pi}\sum_{n=2}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos(2\pi f_c t(2n-1))}_{⑤}$$

(b)



Need: ① $f_c - W > W \iff f_c > 2W$

② $f_c + W < 2f_c \iff f_c > W$

③ $3f_c - W > f_c + W \iff f_c > W$

$$\left.\begin{array}{l} f_c - W > W \iff f_c > 2W \\ f_c + W < 2f_c \iff f_c > W \\ 3f_c - W > f_c + W \iff f_c > W \end{array}\right\} \Rightarrow \underline{\underline{f_c > 2W}}$$