



Fig. 1. $G_T(f)$ versus f under different values of T

We know

$$1 \Rightarrow \int_{-\infty}^{\infty} 1 \cdot e^{-j2\pi ft} dt = \lim_{T \rightarrow \infty} \int_{-T}^T 1 \cdot e^{-j2\pi ft} dt = \lim_{T \rightarrow \infty} G_T(f), \quad (1)$$

where

$$\begin{aligned} G_T(f) &= \int_{-T}^T 1 \cdot e^{-j2\pi ft} dt \\ &= \frac{1}{-j2\pi f} (e^{-j2\pi fT} - e^{j2\pi fT}) \\ &= \frac{1}{-j2\pi f} (-2j) \sin(2\pi fT) \\ &= \frac{\sin(2\pi fT)}{\pi f} = \frac{\sin(2\pi fT)}{2\pi fT} 2T. \end{aligned} \quad (2)$$

Fig.1 is a plot of $G_T(f)$ versus f under different values of T . We can see as T gets larger, the peak of $G_T(f)$ becomes higher. Meanwhile, the mainlobe of $G_T(f)$ which is centered at $f = 0$ becomes more squeezed, and the difference between mainlobe peak and other sidelobe peaks becomes larger also. Hence it is not hard to imagine that as T goes to ∞ , $G_T(f)$ goes to $\delta(f)$.