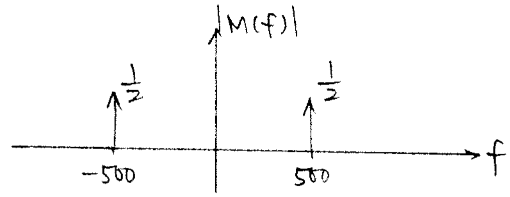
HW #4 Solutions

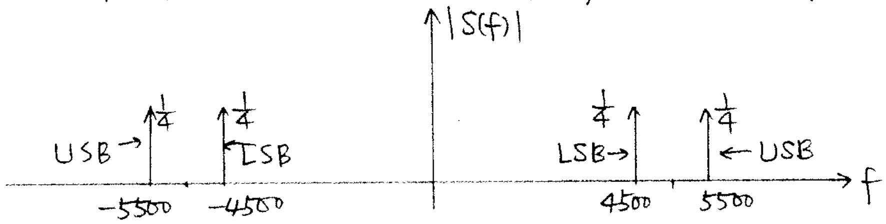
1. $\boxed{4.2\text{-}1}$ (i) $m(t) = \cos(1000\pi t) = \cos(2\pi 500 t)$

$\Rightarrow M(f) = \frac{1}{2}\delta(f-500) + \frac{1}{2}\delta(f+500)$



$S(t) = m(t)\cos(2\pi 5000 t)$

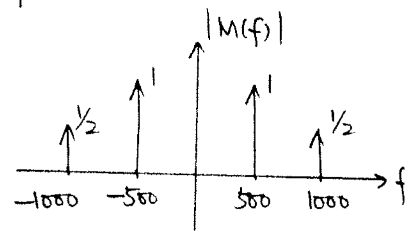
$\Rightarrow S(f) = \frac{1}{4}\delta(f-5500) + \frac{1}{4}\delta(f-4500) + \frac{1}{4}\delta(f+4500) + \frac{1}{4}\delta(f+5500)$

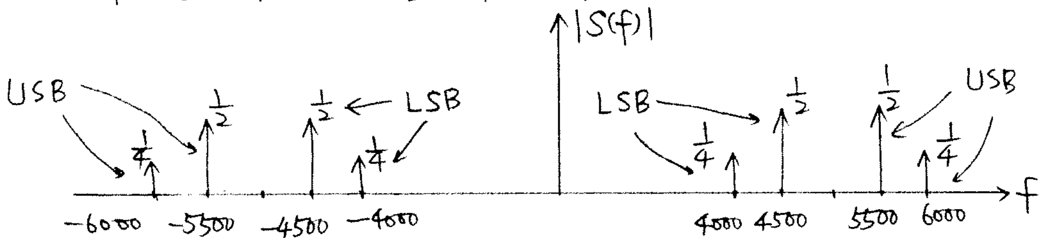


(ii) $m(t) = 2\cos(1000\pi t) + \sin(2000\pi t)$

$\Rightarrow M(f) = \delta(f-500) + \delta(f+500) + \frac{1}{2j}\delta(f-1000) - \frac{1}{2j}\delta(f+1000)$

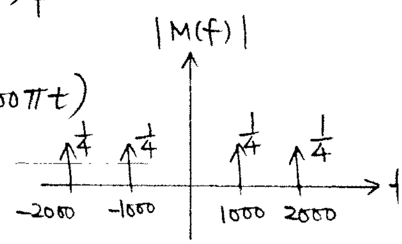
$S(f) = \frac{1}{2}M(f-5000) + \frac{1}{2}M(f+5000)$

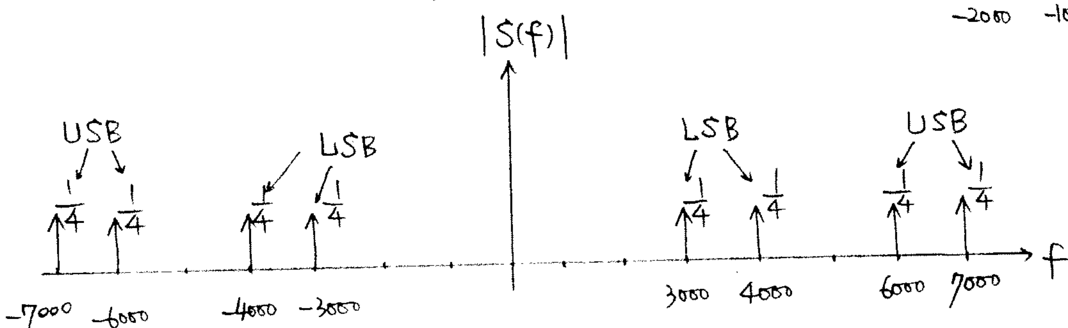




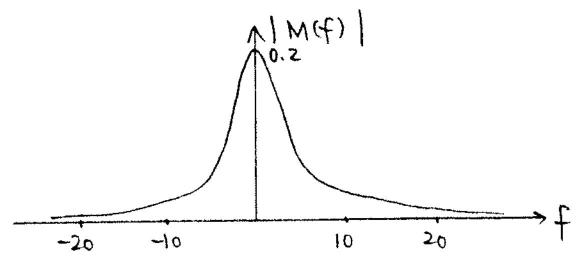
(iii) $m(t) = \cos(1000\pi t)\cos(3000\pi t) = \frac{1}{2}\cos(4000\pi t) + \frac{1}{2}\cos(2000\pi t)$

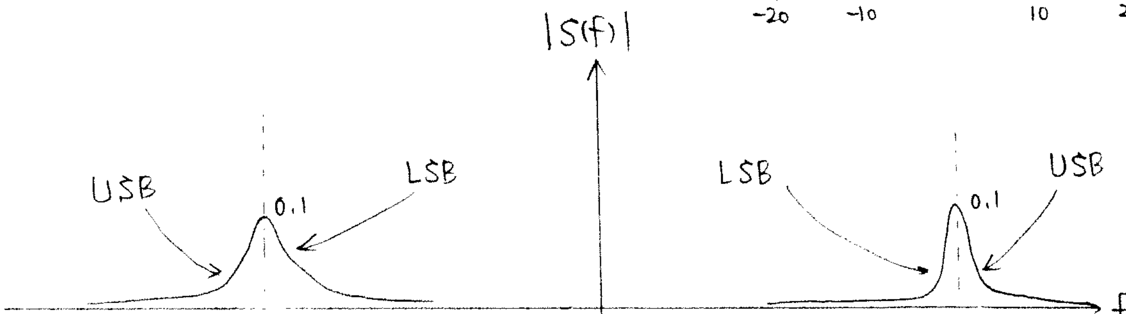
$M(f) = \frac{1}{4}\delta(f-2000) + \frac{1}{4}\delta(f+2000) + \frac{1}{4}(f-1000) + \frac{1}{4}\delta(f+1000)$





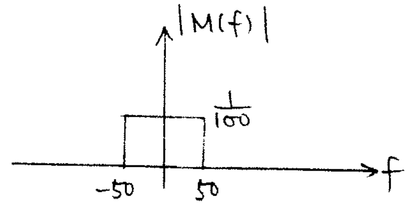
(iv) $m(t) = \exp(-10|t|) \Rightarrow M(f) = \dfrac{20}{100 + 4\pi^2 f^2}$



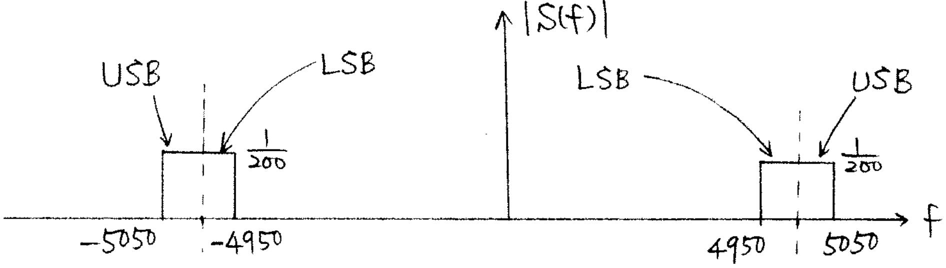


2. $\boxed{4.2\text{-}2}$

(i) $m(t) = \operatorname{sinc}(100\pi t) \Rightarrow M(f) = \frac{1}{100}\operatorname{rect}\left(\frac{f}{100}\right)$



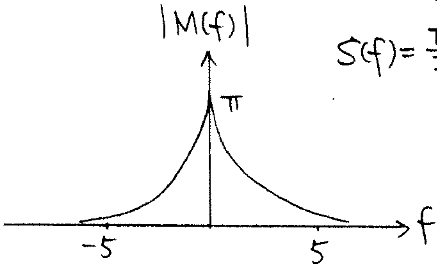
$$S(f) = \frac{1}{200}\operatorname{rect}\left(\frac{f-5000}{100}\right) + \frac{1}{200}\operatorname{rect}\left(\frac{f+5000}{100}\right)$$

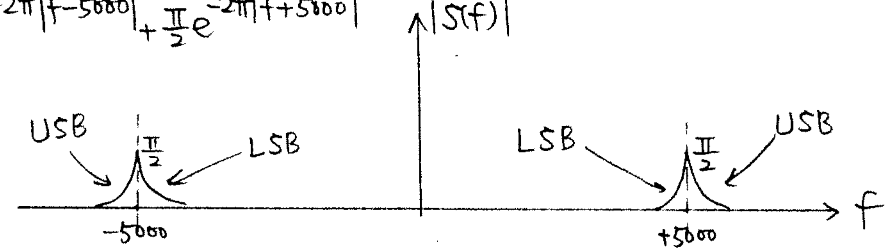


(ii) $m(t) = \frac{1}{1+t^2} \Rightarrow M(f) = \pi e^{-2\pi|f|}$

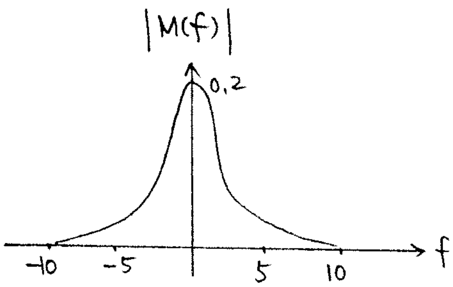
$$e^{-a|t|} \rightleftharpoons \frac{2a}{a^2+4\pi^2 f^2} \Rightarrow \frac{2a}{a^2+4\pi^2 t^2} \rightleftharpoons e^{-a|f|} \underset{a=2\pi}{\Rightarrow} \frac{1}{\pi}\cdot\frac{1}{1+t^2} \rightleftharpoons e^{-2\pi|f|}$$

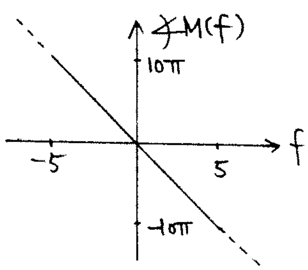
$$S(f) = \frac{\pi}{2}e^{-2\pi|f-5000|} + \frac{\pi}{2}e^{-2\pi|f+5000|}$$

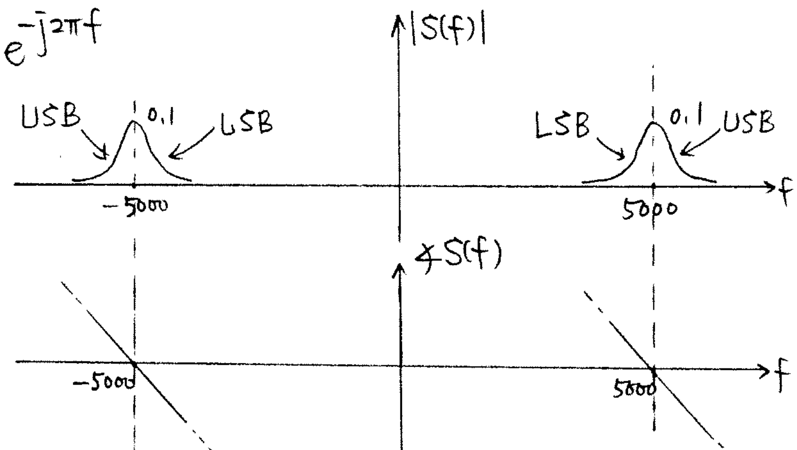




(iii) $m(t) = e^{-10|t-1|} \Rightarrow M(f) = \frac{20}{100+4\pi^2 f^2}\, e^{-j2\pi f}$

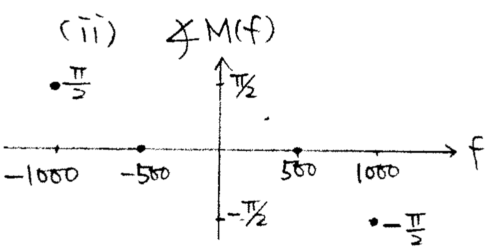


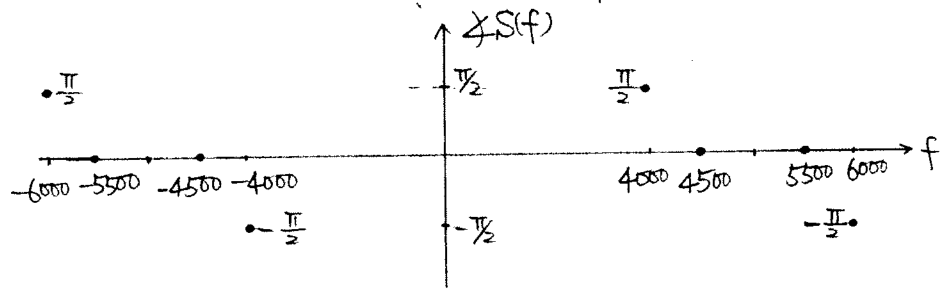




1. $\boxed{4.2\text{-}1}$ continuation of (ii)

(ii)





3. $\boxed{4.2\text{-}3}$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(a) $\cos^3 w_c t = \frac{1}{4}\cos(3w_c t) + \frac{3}{4}\cos(w_c t)$, let $w_c = 2\pi f_c$

An ideal (or constant gain) bandpass filter centered at $\pm f_c$ with bandwidth $2B$ is required to remove $\frac{1}{4}\cos(3w_c t)$ term.

(b) ⓑ: $\frac{1}{8}M(f-3f_c) + \frac{1}{8}M(f+3f_c) + \frac{3}{8}M(f-f_c) + \frac{3}{8}M(f+f_c)$

ⓒ: $\frac{3}{8}M(f-f_c) + \frac{3}{8}M(f+f_c)$, assume ideal (unit gain) bandpass filter is used

Frequency band occupied:

ⓑ: $[-3f_c-B, -3f_c+B]$, $[-f_c-B, -f_c+B]$, $[f_c-B, f_c+B]$, $[3f_c-B, 3f_c+B]$

ⓒ: $[-f_c-B, -f_c+B]$, $[f_c-B, f_c+B]$

(c) $\begin{cases} f_c - B \geq 0 \\ 3f_c - B \geq f_c + B \end{cases} \Rightarrow f_c \geq B \Rightarrow$ minimum usable value of $w_c = 2\pi B$

(d) $\sin^3(w_c t) = \frac{3}{4}\sin(w_c t) - \frac{1}{4}\sin(3w_c t) = \frac{3}{4}\cos(w_c t - \frac{\pi}{2}) - \frac{1}{4}\cos(3w_c t - \frac{\pi}{2})$

Yes, if ideal (unit gain) bandpass filter described in (a) is used, the output (i.e., at ⓒ) is $\frac{3}{4}m(t)\cos(w_c t - \frac{\pi}{2})$, $k = \frac{3}{4}$ and $\theta = \frac{-\pi}{2}$ in this case

(e) This scheme works only for odd n, not for even n.

For example, when $n=2$ we have $\cos^2 w_c t = \frac{1}{2} + \frac{1}{2}\cos(2w_c t)$,

in this case, ⓑ: $\frac{1}{2}M(f) + \frac{1}{4}M(f-2f_c) + \frac{1}{4}M(f+2f_c)$ and we cannot get $k\,m(t)\cos(w_c t + \theta)$ at the output.

4. $\boxed{4.2\text{-}4}$

(a) We use the ring modulator of Figure 4.6a with the carrier frequency $f_1 = 100\text{ kHz}$ ($w_1 = 2\pi\times 10^5$ rad/s) and output bandpass filter centered at $f_c = 5f_1 = 500\text{ kHz}$. The output $v_i(t)$ (by (4.7b)) is:

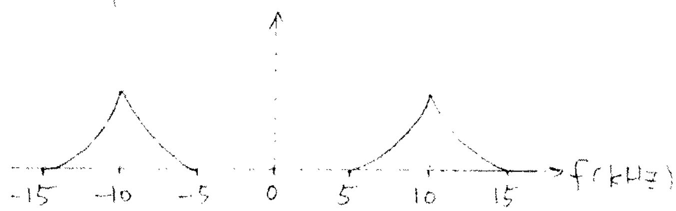
$$v_i(t) = \frac{4}{\pi}\left[m(t)\cos(w_1 t) - \frac{1}{3}m(t)\cos(3w_1 t) + \frac{1}{5}m(t)\cos(5w_1 t) - \cdots\cdots\right]$$

The output bandpass filter suppresses all terms except the one centered at 500 kHz ($f_c = 5f_1$). Thus the desired signal $y(t) = \frac{4}{5\pi}m(t)\cos(5w_1 t)$

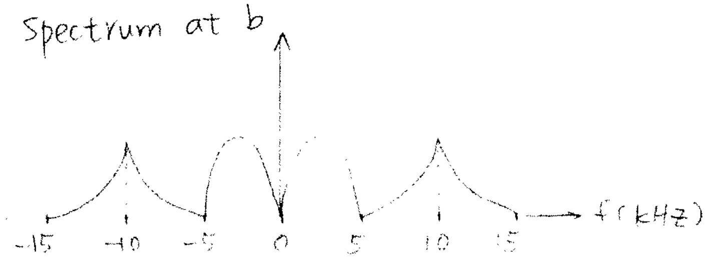
(b) $y(t) = k\,m(t)\cos(w_c t) = \frac{4}{5\pi}m(t)\cos(5w_1 t) \Rightarrow k = \frac{4}{5\pi}$

5. $\boxed{4.2\text{-}7}$

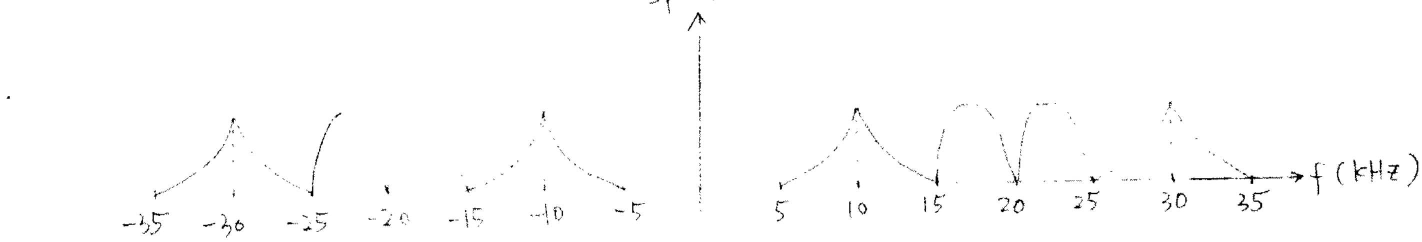
(a) Spectrum at a



Spectrum at b

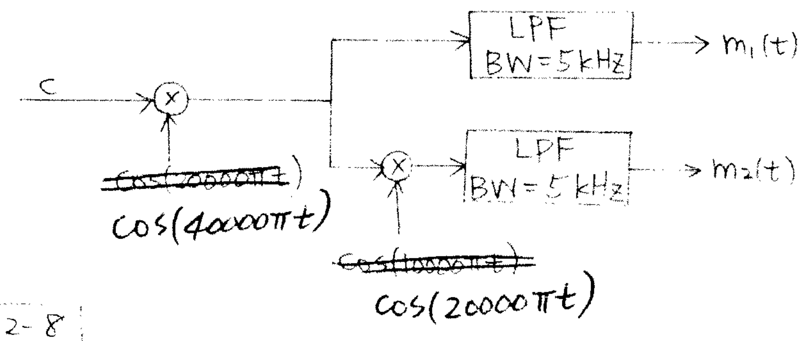


Spectrum at c



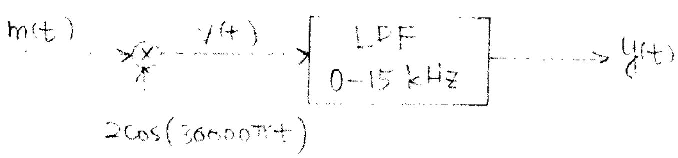
(b) From spectrum at c, the distortionless channel bandwidth must be at least 30000 Hz (i.e., 35k − 5k = 30k Hz)

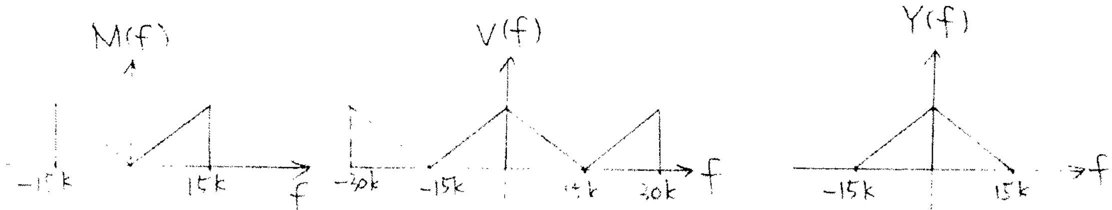
(c)



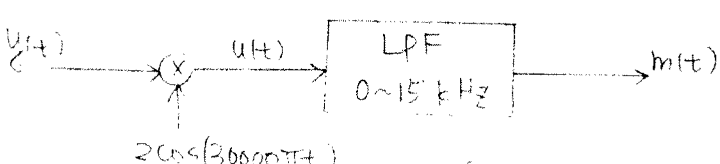
6. $\boxed{4.2\text{-}8}$

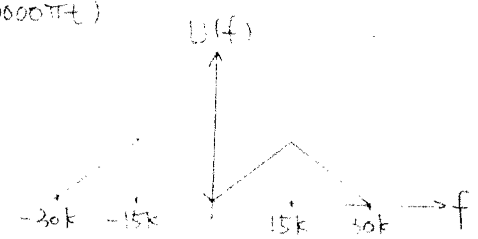
(a)





(b) Let y(t) pass through the same scrambler, the output will be m(t)

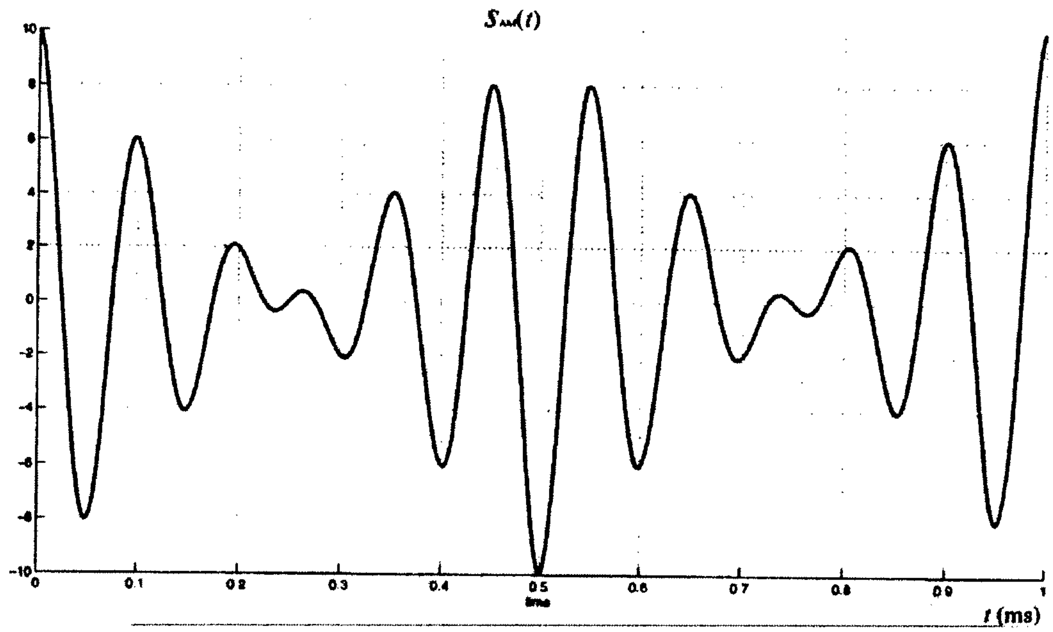




7. $\boxed{4.3-4}$

(a) $S_{AM}(t) = m(t)\cos(2\pi f_c t)$

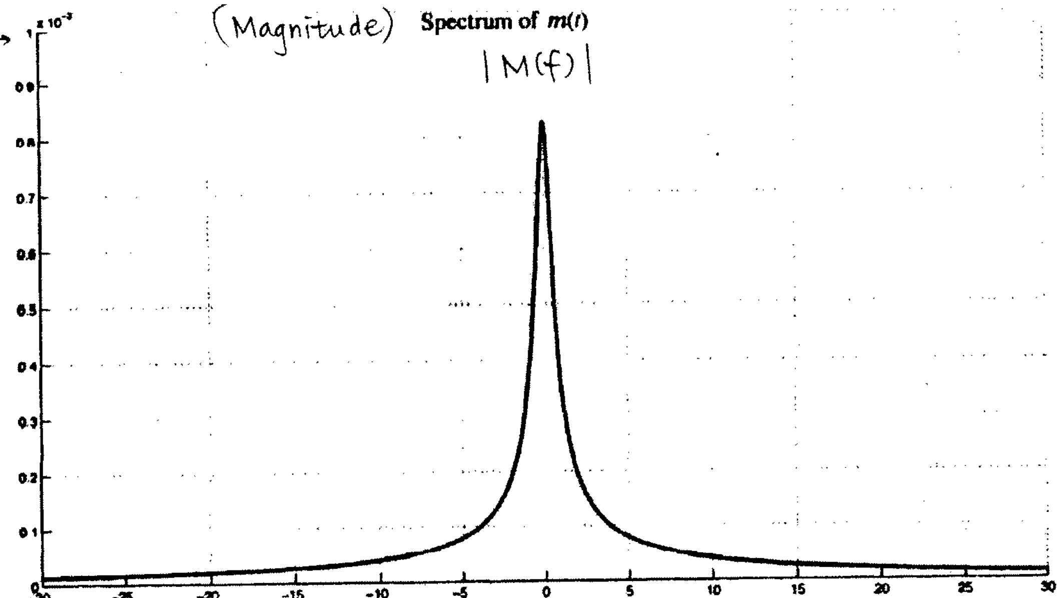
The following figure shows the case when $f_c = 10\,kHz$.



(b) From section 4.3 of textbook (P.191~P.192), we know the envelope of $m(t)\cos(\omega_c t)$ is $|m(t)|$, under the assumption that $m(t)$ varies slowly in comparison with the carrier $\cos(\omega_c t)$. Hence, it is clear that the envelope of $(A+m(t))\cos(\omega_c t)$ is $|A+m(t)|$. Now, if $A+m(t)>0, \forall t$, then $A+m(t) = |A+m(t)|$. Therefore, the condition for demodulating an AM signal using envelope detector is $A+m(t)>0, \forall t$.

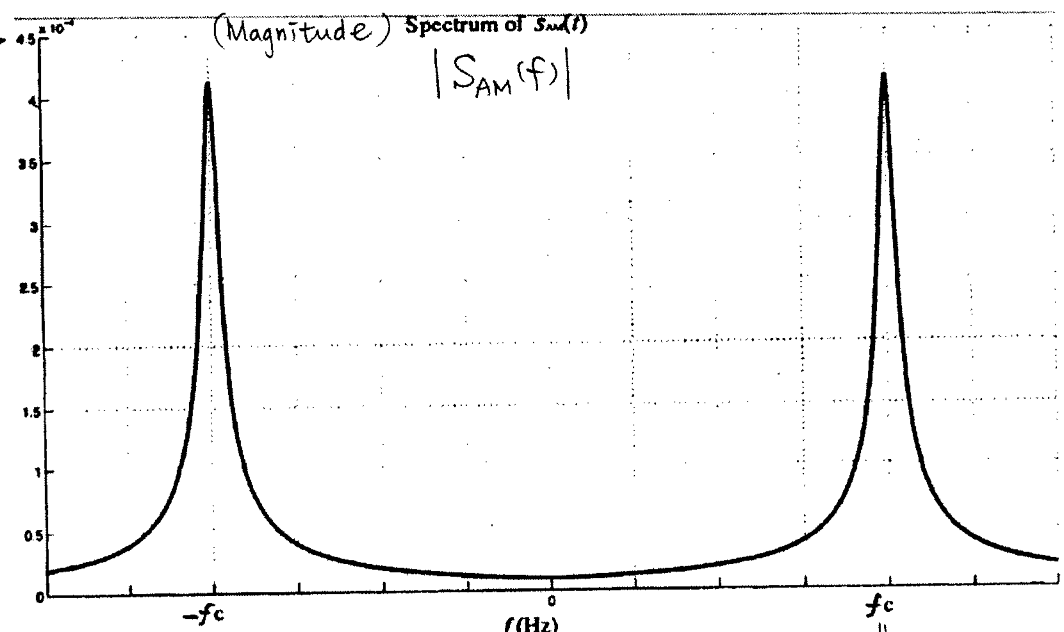
8. $\boxed{4.3\text{-}5}$

(a) $m(t) = e^{-3t}u(t-2) = e^{-6}\cdot e^{-3(t-2)}u(t-2) \underset{F^{-1}}{\overset{F}{\rightleftharpoons}} \frac{e^{-6}}{3+j2\pi f}e^{-j2\pi f2} = \frac{e^{-6}}{3+j2\pi f}e^{-j4\pi f} = M(f)$



(b) $S_{AM}(t) = m(t)\cos(2\pi f_c t) \underset{F^{-1}}{\overset{F}{\rightleftharpoons}} S_{AM}(f) = \frac{1}{2}M(f-f_c)+\frac{1}{2}M(f+f_c)$, $f_c = 1000\,Hz$

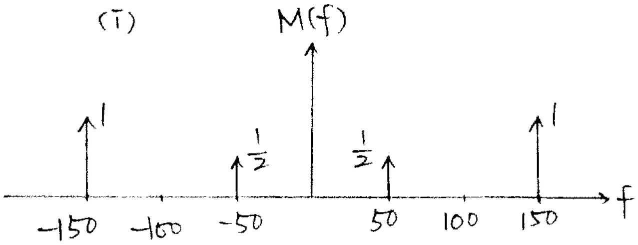
$$= \frac{0.5e^{-6}}{3+j2\pi(f-1000)}e^{-j4\pi f} + \frac{0.5e^{-6}}{3+j2\pi(f+1000)}e^{-j4\pi f}$$



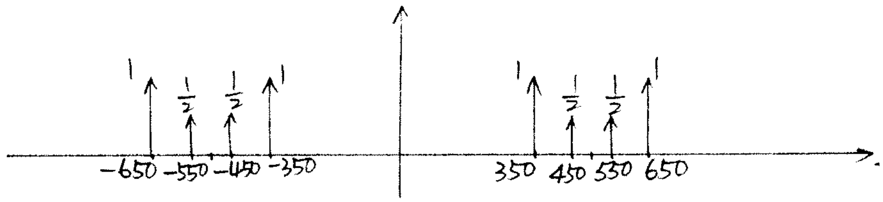
9. $\boxed{4.4\text{-}2}$

(a) $m(t)=\cos(100\pi t)+2\cos(300\pi t) \Rightarrow M(f)=\frac{1}{2}\delta(f-50)+\frac{1}{2}\delta(f+50)+\delta(f-150)+\delta(f+150)$

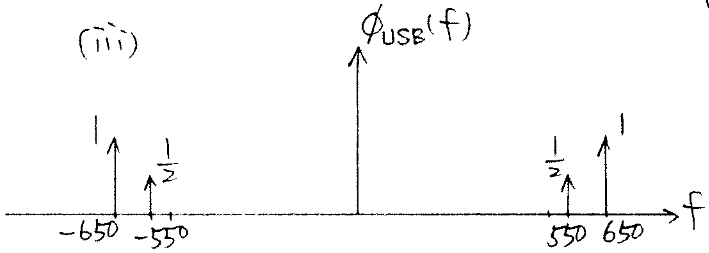
(i)



(ii) $s(t)=2m(t)\cos(1000\pi t) \Rightarrow S(f)=M(f-500)+M(f+500)$



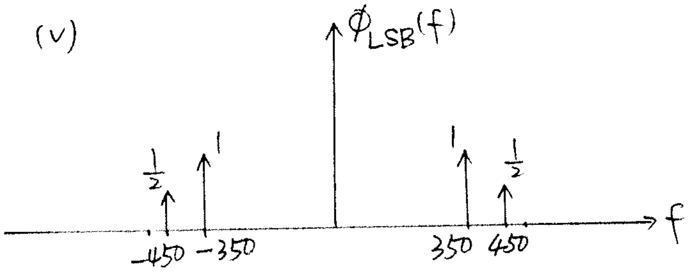
(iii)



(iv)

$$\varphi_{USB}(t)=F^{-1}(\Phi_{USB}(f))$$
$$=\cos(2\pi(550)t)+2\cos(2\pi(650)t)$$
$$=\cos(1100\pi t)+2\cos(1300\pi t)$$

(v)

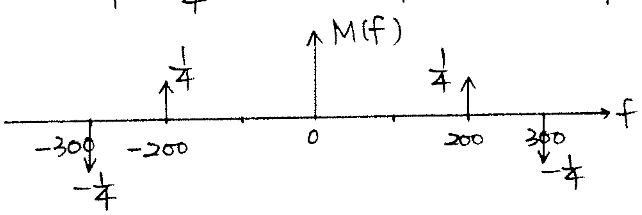


(v)

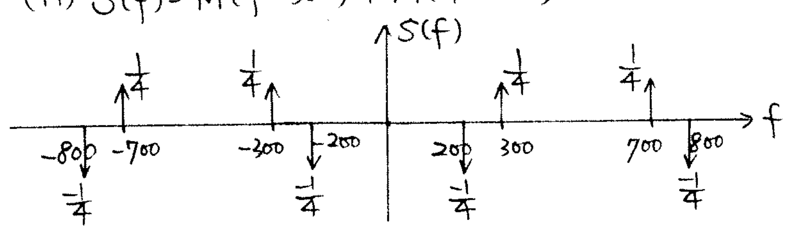
$$\varphi_{LSB}(t)=F^{-1}(\Phi_{LSB}(f))$$
$$=\cos(2\pi(450)t)+2\cos(2\pi(350)t)$$
$$=\cos(900\pi t)+2\cos(700\pi t)$$

(b) $m(t)=\sin(100\pi t)\sin(500\pi t)=\frac{1}{2}\cos(400\pi t)-\frac{1}{2}\cos(600\pi t)$

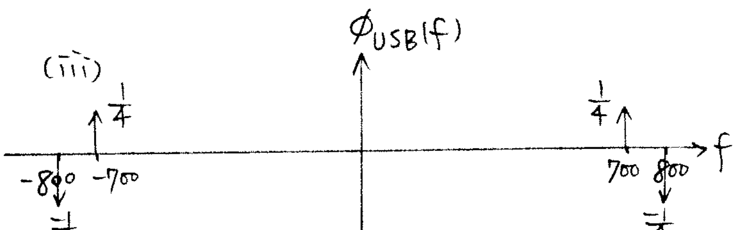
(i) $M(f)=\frac{1}{4}\delta(f-200)+\frac{1}{4}\delta(f+200)-\frac{1}{4}\delta(f-300)-\frac{1}{4}\delta(f+300)$



(ii) $S(f)=M(f-500)+M(f+500)$



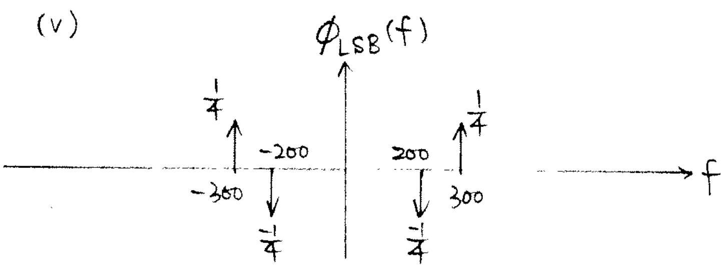
(iii)



(iv) $\varphi_{USB}(t)=F^{-1}(\Phi_{USB}(f))$

$$=\frac{1}{2}\cos(2\pi(700)t)-\frac{1}{2}\cos(2\pi(800)t)$$
$$=\frac{1}{2}\cos(1400\pi t)-\frac{1}{2}\cos(1600\pi t)$$

(v)



(v) $\varphi_{LSB}(t)=F^{-1}(\Phi_{LSB}(f))$

$$=\frac{1}{2}\cos(2\pi(300)t)-\frac{1}{2}\cos(2\pi(200)t)$$
$$=\frac{1}{2}\cos(600\pi t)-\frac{1}{2}\cos(400\pi t)$$

10. $\boxed{4.4\text{-}4}$

$\mathrm{sinc}(\pi t) \rightleftharpoons \mathrm{rect}(f)$

$\Rightarrow \mathrm{sinc}^2(\pi t) \rightleftharpoons \Delta\left(\frac{f}{2}\right) \Rightarrow \mathrm{sinc}^2(2\pi Bt) \rightleftharpoons \frac{1}{2B}\Delta\left(\frac{f}{4B}\right) \Rightarrow \pi B\,\mathrm{sinc}^2(2\pi Bt) \rightleftharpoons \frac{\pi}{2}\Delta\left(\frac{f}{4B}\right)$

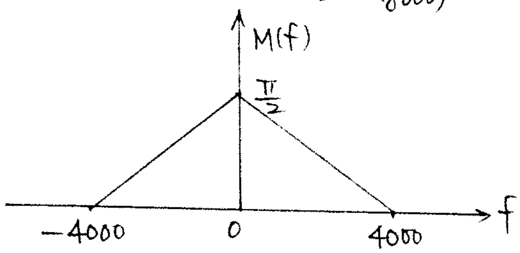
Hence $m(t) = \pi B\,\mathrm{sinc}^2(2\pi Bt) \underset{F^{-1}}{\overset{F}{\rightleftharpoons}} M(f) = \frac{\pi}{2}\Delta\left(\frac{f}{4B}\right)$, where $\Delta\left(\frac{f}{2}\right) = \begin{cases} 1-|f|, & \text{if } |f| \le 1 \\ 0, & \text{else.} \end{cases}$

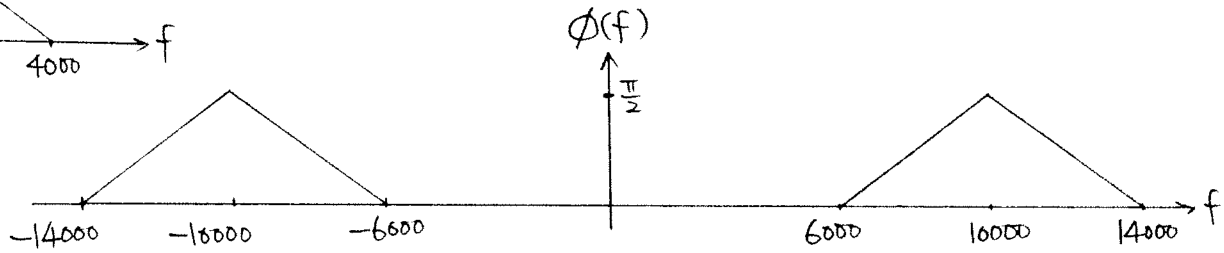
(a) $M(f) = \frac{\pi}{2}\Delta\left(\frac{f}{8000}\right)$

The DSB-SC signal $\varphi(t) = m(t)\,2\cos(\omega_c t)$

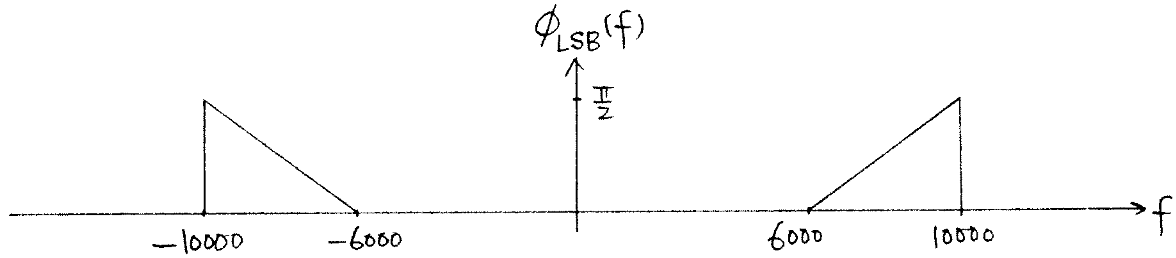
$\Rightarrow \Phi(f) = F(\varphi(t)) = M(f-f_c) + M(f+f_c)$,

$= M(f-10^4) + M(f+10^4)$





(b)



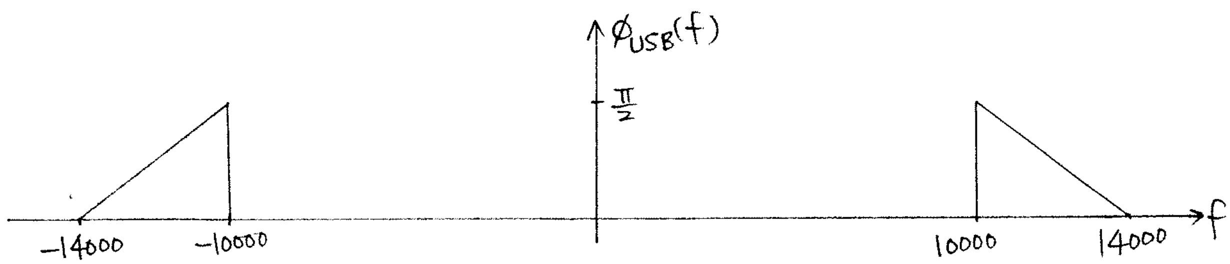
(c) For LSB:

$$\varphi_{LSB}(t) = \int_{-\infty}^{\infty} \Phi_{LSB}(f) e^{j2\pi ft} df = 2\int_{6000}^{10000} \frac{\pi}{8000}(f-6000)\cos(2\pi ft)\,df$$

$$= \frac{\pi}{4000}\int_{6000}^{10000} f\cos(2\pi ft)\,df - \frac{3\pi}{2}\int_{6000}^{10000}\cos(2\pi ft)\,df$$

$$= \frac{1}{2t}\sin(20000\pi t) + \frac{1}{16000\pi t^2}\cos(20000\pi t) - \frac{1}{16000\pi t^2}\cos(12000\pi t)$$

(c) For USB:



$$\varphi_{USB}(t) = \int_{-\infty}^{\infty} \Phi_{USB}(f) e^{j2\pi ft} df = 2\int_{10000}^{14000} \frac{\pi}{8000}(14000-f)\cos(2\pi ft)\,df$$

$$= \frac{-\pi}{4000}\int_{10000}^{14000} f\cos(2\pi ft)\,df + \frac{7\pi}{2}\int_{10000}^{14000}\cos(2\pi ft)\,df$$

$$= -\frac{1}{2t}\sin(20000\pi t) - \frac{1}{16000\pi t^2}\cos(28000\pi t) + \frac{1}{16000\pi t^2}\cos(20000\pi t)$$