COMMUNICATION SYSTEMS

HOMEWORK # 5:

Please work out the **ten** (10) problems stated below – LD refers to the text: B,P. Lathi and Z. Ding, *Modern Digital and Analog Communication Systems* (Fourth Edition), Oxford University Press, Oxford (UK), 2009. Exercise **2.1-1** (LD) refers to Exercise 1 for Section 2.1 of LD.

Show work and **explain** reasoning. Three (3) problems, selected at random amongst these ten problems, will be marked.

1.

Problem **4.3-7** (LD).

2.

Problem **4.3-8** (LD).

3.

Problem **4.3-9** (LD).

4.

Problem **4.4-1** (LD).

5.

Consider a signal $g : \mathbb{R} \to \mathbb{R}$ such that

$$\int_{\mathbb{R}} |g(t)| dt < \infty \quad \text{and} \quad \int_{\mathbb{R}} |g(t)|^2 dt < \infty.$$

Explain how to make sense of the following statement: The Hilbert transform $g_h : \mathbb{R} \to \mathbb{R}$ of g is well defined whenever we have

$$\int_{\mathbb{R}} |G(f)| df < \infty.$$

and in the process give an expression for g_h. Compute the energy

$$\int_{\mathbb{R}} |g_h(t)|^2 dt,$$

and show that g and g_h are orthogonal signals in the sense that

$$\int_{\mathbb{R}} g(t)g_h(t)dt = 0.$$

6.

Problem **4.4-3** (LD).

7.

Consider the quadrature filter with frequency response

$$H(f) = \begin{cases} e^{-j\frac{\pi}{2}} & \text{if } f > 0 \\ 0 & \text{if } f = 0 \\ e^{j\frac{\pi}{2}} & \text{if } f < 0 \,. \end{cases}$$

This filter can used to implement the Hilbert transform.

We can generalize this concept to a new transform which introduces a phase shift of θ in the frequency component. Thus, with

$$H_\theta(f) = \begin{cases} e^{-j\theta} & \text{if } f > 0 \\ 0 & \text{if } f = 0 \\ e^{j\theta} & \text{if } f < 0 \,. \end{cases}$$

we introduce the θ-transform of the signal $g : \mathbb{R} \to \mathbb{R}$ as the signal $g_\theta : \mathbb{R} \to \mathbb{R}$ determined through

$$G_\theta(f) = H_\theta(f)G(f), \quad f \in \mathbb{R}.$$

As in Problem **5**, give conditions to ensure that g_θ is well defined. Under these conditions show that g_θ is a linear combination of g and of its Hilbert transform g_h.

8.

Problem **4.4-5** (LD).

9.

Problem **4.4-6** (LD).

10.

Problem **4.4-7** (LD).