COMMUNICATION SYSTEMS

ADDITIONAL PROBLEMS:

Consider the information-bearing signal $g : \mathbb{R} \to \mathbb{R}$ given by

$$g(t) = \frac{\sin \pi t}{\pi t}, \quad t \in \mathbb{R}.$$

a. Find the Fourier transform $G(f)$ of g. Is this signal band-limited? If so find its Nyquist rate $f^\star$.

Next, we sample the signal g periodically at times $k = 0, \pm T_s, \pm 2T_s, \ldots$ for some $T_s > 0$. The resulting samples are used to generate a PAM signal $g_{\text{PAM}} : \mathbb{R} \to \mathbb{R}$ given by

$$g_{\text{PAM}}(t) = \sum_{k=-\infty}^{\infty} g(kT_s) \cdot p(t - kT_s), \quad t \in \mathbb{R}$$

where the rectangular pulse $p : \mathbb{R} \to \mathbb{R}$ is defined by

$$p(t) = \begin{cases} 1 & \text{if } \ 0 \le t \le \tau \\ \\ 0 & \text{otherwise} \end{cases}$$

for some $\tau > 0$.

b. Derive an expression for the Fourier transform $G_{\text{PAM}}(f)$ of g_{PAM}. Sketch $|G_{\text{PAM}}(f)|$ as a function of f when $T_s = \frac{2}{3}$ and $\tau = \frac{1}{3}$.

c. Usually the original information-bearing signal g is recovered from g_{PAM} through a series of steps: The PAM signal g_{PAM} is first passed through a low-pass filter, and the resulting output signal then "equalized" in order to compensate for amplitude distortions. Propose one such low-pass filter[1] and determine the maximum amplitude distortion.

In this problem, we consider a system where voice samples are expected to assume values in the symmetric interval $[-256, 256]$. These voice samples are quantized according to a two-stage non-uniform quantization technique based on the compression characteristic

$$\Phi(m) := \sqrt{|m|} \cdot \text{sgn}(m), \quad m \in \mathbb{R}$$

[1] Please give a specific frequency response function!

followed by a 5-bit uniform quantizer.

a. Describe the cells $C_1, \ldots C_L$ of the uniform quantizer and determine the associated representation values $V_1, \ldots, V_L$. What is the value of L here?

b. Describe the cells $\tilde{C}_1, \ldots \tilde{C}_L$ of the non-uniform quantizer and determine the associated representation values $\tilde{V}_1, \ldots, \tilde{V}_L$. In particular, what is the quantized version of the sample $m = 60$?.

c. Derive the expansion characteristic associated with the compressor Φ given above.

d. In the context of a PCM-like system, is there any advantage in using the compressor with characteristic

$$\Psi(m) := 16\sqrt{|m|} \cdot \mathrm{sgn}(m) = 16\Phi(m), \quad m \in \mathbb{R}$$

instead of the one introduced earlier. Explain your answer.

Consider a $(7, 4)$ linear block code in systematic form described by

$$\begin{aligned} c_1 &\equiv m_1 \\ c_2 &\equiv m_2 \\ c_3 &\equiv m_3 \\ c_4 &\equiv m_4 \\ c_5 &\equiv m_1 + m_2 + m_4 \\ c_6 &\equiv m_1 + m_2 + m_3 \\ c_7 &\equiv m_1 + m_3 + m_4 \end{aligned}$$

a. List all the codewords for this code. Please use the table below.

b. Determine the parity matrix $\boldsymbol{P}$ for this code.

c. Determine the check matrix $\boldsymbol{H}$ for this code.

d. Compute the Hamming distance of this code and discuss its generic capabilities for error detection and correction.

e. Let $\mathbf{c}' = (1, 0, 1, 0, 1, 0, 1)$ denote vector obtained by the receiver upon demodulating the (portion of the) received signal associated with the codeword $\mathbf{c}$. Compute the syndrome of $\mathbf{c}'$ and decide from this calculation whether $\mathbf{c}'$ is a valid codeword.

f. Find the nearest neighbor estimate $\hat{\mathbf{c}}$ of $\mathbf{c}'$. Is it the case that $\hat{\mathbf{c}}$ was indeed the codeword sent? Explain your answer!

The signal $c : \mathbb{R} \to \mathbb{R}$ is periodic with period $2T$; on the interval $[-T, T]$ it is given by

$$c(t) = A\left(1 - \frac{|t|}{T}\right), \quad |t| \leq T$$

with $A > 0$.

a. Evaluate the Fourier coefficients for $c : \mathbb{R} \to \mathbb{R}$

b. Show that the Fourier series expansion of the signal c is of the form

$$c(t) = \sum_{k=0}^{\infty} \gamma_k \cos\left(\pi \frac{k}{T} t\right), \quad t \in \mathbb{R}$$

for a suitable sequence of coefficients $\{\gamma_k,\ k = 0, 1, \ldots\}$. Give a closed-form expression for each of the coefficients $\{\gamma_k,\ k = 0, 1, \ldots\}$.

c. Evaluate the infinite series

$$\sum_{k=1}^{\infty} \gamma_k^2.$$

The information-bearing signal $m : \mathbb{R} \to \mathbb{R}$ is a low-pass signal with cutoff frequency $B > 0$. With $A > 0$, the signal $x : \mathbb{R} \to \mathbb{R}$ defined by

$$x(t) = m(t) + A\cos\left(2\pi f_c t\right), \quad t \in \mathbb{R}$$

is applied to a non-linear device whose output $y : \mathbb{R} \to \mathbb{R}$ is given by

$$y(t) = x(t) + \frac{1}{2}x(t)^2, \quad t \in \mathbb{R}.$$

a. Sketch the spectrum of the output signal $y : \mathbb{R} \to \mathbb{R}$. Clearly identify its various components.

b. Explain how using only linear filtering operations, the signal $y : \mathbb{R} \to \mathbb{R}$ can be processed to produce the AM signal $y_{\text{AM}} : \mathbb{R} \to \mathbb{R}$ given by

$$y_{\text{AM}} = (1 + m(t))\cos\left(2\pi f_c t\right), \quad t \in \mathbb{R}.$$

Give a block diagram of your proposed implementation. Carefully explain your answer!

c. In particular, give conditions on the parameters B and f_c in order for the proposed implementation to work?
