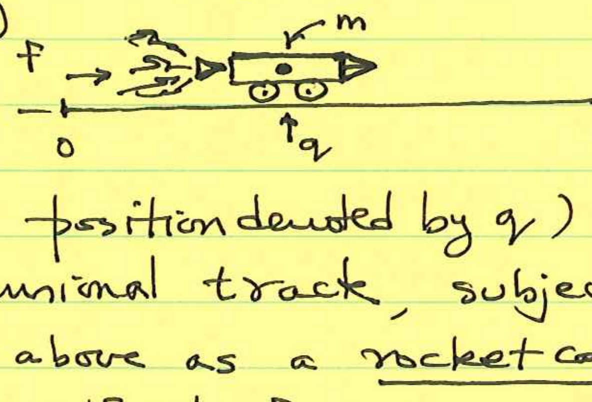
ENEE 460 Control Systems <u>Lecture 1</u>

Physical principles and models are central to understanding nature. Control systems enable physical systems to re-shape their nominal behavior to meet some <u>purpose</u>, e.g. balance a broom (or SEGWAY®), break crude oil into gasoline in refineries, achieve interplanetary flight, automate assembly lines by precise control of processes such as welding, etc.

Physical systems of interest may be well-understood for us to enable accurate models. Knowledge of model allows us to build (design and implement) control systems that achieve a purpose with precision. In the absence of good models, one can still find sound <u>principles</u> for control.

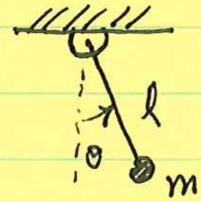
Physical models for us are grounded in Newton's laws:



(1) $$m\ddot{q} = f$$

for a particle (with position denoted by q) moving on a 1-dimensional track, subject to force f, pictured above as a <u>rocket car</u>; simple <u>pendulum</u>, swinging <u>freely</u> from a <u>frictionless</u> pivot of mass (of pendulum bob) m, effective length l satisfying

$$m l^2 \ddot{\theta} + m g l \sin(\theta) = 0$$



equivalently

(2) $$\ddot{\theta} + \frac{g}{l} \sin(\theta) = 0$$

where g = acceleration due to gravity.

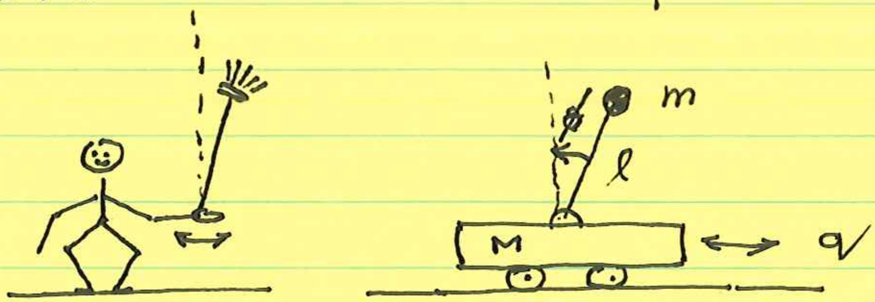
When the rocket car experiences real resistance (friction at wheels, air drag) and runs out of fuel it will come to an eventual stop; similarly the unforced pendulum with pivot resistance will come to a stop.

But, if pendulum represents a child on a garden swing, control can be exerted by pumping action — child gets up and sits down to continually alter the effective length of the pendulum by altering the location of the center of mass:

$$\ddot{\theta} + \frac{g}{l(t)} \sin(\theta) = 0$$

A child learns to modulate $l(\cdot)$.

A similar picture $\phi = \pi - \theta$



man balancing a broom vertically on the palm outstretched and moving horizontally back and forth; inverted pendulum on a cart of mass M (this latter illustration is a coupling of (1) and (2) in essence – what are the system equations?)

Here we are clearly actively controlling a system to meet a purpose – balancing. The cart motion has to be designed to achieve control. Keeping track of both θ and q means that this system has 2 degrees of freedom (DOF) as compared to the 1 DOF systems in equations (1) and (2). (The "Human Transporter" SEGWAY is a sophisticated version of this; Dean Kamen's original patent and company are now owned by NineBot.)

Modify the pendulum equation (2) to account for friction at the pivot, leading to

$$\underset{\uparrow\ b = \text{damping factor}}{}$$

(3) $$\ddot{\theta} + b\dot{\theta} + \frac{g}{l}\sin(\theta) = 0$$

where $b > 0$. What is the resulting behavior? Consider the expression

$$H = \frac{\dot{\theta}^2}{2} - \frac{g}{l}\cos(\theta)$$

This can be interpreted, up to a scale factor, as the total energy of the pendulum = kinetic energy + potential energy.

Under the dynamics (3),

$$\frac{dH}{dt} = \frac{\partial H}{\partial \theta}\dot{\theta} + \frac{\partial H}{\partial \dot{\theta}}\ddot{\theta}$$

$$= \dot{\theta}\left(\ddot{\theta} + \frac{g}{l}\sin(\theta)\right)$$

$$= -b\dot{\theta}^2 \leq 0$$

Thus H decreases until it can decrease no further (comes to rest), with $\dot{\theta} = 0$. At what rest position? Only two possibilities: $\theta = 0$, and $\theta = \pi$. What we observe is $\theta = 0$, a stable equilibrium.

(i.e about $\theta = \pi$)

The problem of balancing a broom vertically (or the inverted pendulum problem), can be thought of as finding a suitable (control) forcing f so that damped pendulum equation (3) is turned into

$$(4) \qquad \ddot{\theta} + b\dot{\theta} + \frac{g}{l}\sin(\theta) = f,$$

the forced damped pendulum equation.

Let us see what happens to (4) when forcing f is turned off and we are studying behavior near $\theta = \pi$, in other words we consider

$$\theta(t) = \pi + \delta(t).$$

We get,

$$(5) \qquad \ddot{\delta} + b\dot{\delta} + \frac{g}{l}(-\sin\delta) = 0$$

which can be approximated as

$$(5') \qquad \ddot{\delta} + b\dot{\delta} - \frac{g}{l}\delta = 0$$

a linear 2^{nd} order differential equation, when δ is small. We have used $\sin(\delta) \approx \delta$ for δ small.

But will it stay small?

Consider the characteristic polynomial of (5')

$$\chi(s) = \left(s^2 + bs - \frac{g}{\ell}\right).$$

Solutions to (5') have the form

$$\delta(t) = A e^{\lambda_+ t} + B e^{\lambda_- t}$$

where $\lambda_\pm$ are roots of the characteristic polynomial,

$$\lambda_\pm = \frac{-b \pm \sqrt{b^2 + 4g/\ell}}{2}$$

$\lambda_+ > 0$, hence, in general **NO**! it will grow exponentially

With no control forcing the broom/pendulum will fall.

If we put the control back in and set $\underbrace{\boxed{f(t) = -k_p \delta}}$ FEEDBACK on the right hand side of (5') then the solutions are decaying exponentials provided the <u>gain</u> $k_P > \frac{g}{\ell}$

$\Rightarrow \quad \delta(t) \to 0 \qquad$ (stay vertical).

Thus the proportional feedback law $f(t) = -k_P \, \delta(t)$ stabilizes the vertical position provided the gain $k_P > \frac{g}{\ell}$

$$\omega = \sqrt{\frac{g}{\ell}}$$ is natural frequency of oscillation of pendulum about straight down configuration.

· High natural frequency demands <u>high gain</u> k_P