

ENEE 601 MIDTERM SOLUTIONS SPRING 2007

11. At high temperatures the Fermi level goes to mid-gap. This is because the thermally generated electron and hole concentrations overwhelm the dopant-induced mobile charge densities. The material “looks” intrinsic and $n=p$. When $n=p$, the charge-neutrality relationship becomes:

$$n_i \exp\left[\left(\frac{1}{kT}\right)(E_i - E_f)\right] = n_i \exp\left[\left(\frac{1}{kT}\right)(E_f - E_i)\right] \quad (1)$$

(Note: I’ve used the “physics convention” in assigning relative energies.)

In any event, this solves as $E_f = E_i$.

As the temperature lowers, the Fermi distribution function “sharpens up.” It becomes more like a step function. The intrinsic concentration (n_i) starts to “freeze out.” But as long as the thermal energy (kT/q) is higher than the separation of the trap from its populating band edge, the dopant will contribute mobile carriers to the band. Thus, at low temperatures, in order to keep any mobile charges in the band, the probability that the trap is charged must remain significant (at least not infinitesimal).

Consider an n-doped semiconductor. The density of mobile electron carriers will be:

$$n = \int_{E_g} (1 - F(E)) D_d(E) dE \quad (2)$$

where $(1 - F(E))$ is the probability a trap at E is *unoccupied*, and D_d is the density of donor states in the gap. This is just a Dirac delta function: $\delta(E - E_t)$, where E_t is the trap energy in the gap. Taking the degeneracy factor, g , into account:

$$N_d^+ = N_d \left(1 - \frac{1}{1 + \frac{1}{2} \exp\left(\frac{1}{kT}(E_t - E_f)\right)} \right) = \frac{N_d}{1 + 2 \exp\left(-\frac{1}{kT}(E_t - E_f)\right)} \quad (3)$$

This is equation 2.9 in the text. The condition that we have significant ionization reduces to the requirement that the term: $2 \exp\left(-\frac{1}{kT}(E_t - E_f)\right)$ must be small. Thus the trap energy must always be above the Fermi level. This is another way of saying that when the donor trap “sinks” in the Fermi sea, it discharges and becomes neutral. Now, let us look at the other “physical” demands we put on the system. As stated above, as long as the thermal energy of the system is more than the energy necessary to promote the mobile carrier from the trap to the band, we expect significant occupancy - an occupancy

determined only by the trap density. Thus, we'd expect the argument of the exponent to be fairly constant:

$$(E_t - E_f) = CkT \quad (4)$$

where C is a constant. The only way for this to happen is for The energy difference on the left hand side of this equation to go to zero as the temperature goes to zero. Thus, $E_t \rightarrow E_f$ as the temperature goes to zero.

I2. We are interested in finding the ratio N_d^+/N_d . We are given that the Fermi level is 0.35V above midgap. Assume the trap is very close to the conduction band edge (a shallow trap, characteristic of most donor dopants). from equation 3, we find this ratio to be: 0.999997. You say: well you didn't give the trap energy. In the problem here, I took the trap as being up against the band edge. How much of an error would that make? A "large" separation for a shallow dopant would be 30 mV. This would make the efficiency 0.999999. just a tad closer to unity.

III1. As we said in class, whenever we put a metal in contact with a semiconductor, the work-function difference creates a space charge. This space charge is usually a depletion layer. The Depletion layer creates a barrier to majority charge transport. Heavily doping the semiconductor surface alleviates this problem in two ways. First, the depletion thickness goes inversely as the square-root of the doping, Thus, the heavily doped surface has a very shallow depletion. The depletion becomes so shallow, it is "tunnelable" at low voltages. Also, the depletion fields get very high and any applied bias causes an electrostatic breakdown, effectively eliminating the barrier.

II2. While there is a depletion associated with the metal-semiconductor contact (which we try to minimize by heavy doping), there is no depletion associated with the doping gradients in either the p+/p or n+/n structures. But the gradient does cause a "displacement" of mobile majority charge in a direction of lower concentration. This diffusive transport "shuts off" when the electrostatic field generated by the charge displacement counterbalances the tendency to diffuse. So, consider the n+/n case in which the dopant concentration lowers as we move along the positive x axis. The mobile electrons can be viewed as a "lattice" that displaces a bit to the right with respect to the fixed dopant ion lattice. The electron lattice is negative, the ion lattice is positive. Thus. the "quasi-neutral field points right. The same principle holes for the p+/p structure. Only now, the hole lattice is displaced a bit to the right and the E-field vector points left.

IIIa. The CV plots are shown below. I know there was some ambiguity in the question on the issue of the dopant type. I just asked for consistency!

IIIb1. $C_{max} = C_{ox} = \frac{\epsilon_{SiO_2}}{d_{ox}} = \frac{3.45 \times 10^{-13}}{10^{-6}} = 3.45 \times 10^{-7} \text{Fd/cm}^2$. C_{max} occurs whenever the surface is accumulated (at negative voltages for an n-channel and at positive voltages for a p-channel) and inversion at low frequencies.

IIIb2. $C_{min} = \frac{C_{ox}C_{sc}(\phi_s=2\phi_b)}{C_{ox}+C_{sc}(\phi_s=2\phi_b)}$. $C_{sc}(\phi_s = 2\phi_b) = \frac{\epsilon_{si}}{x_{sc}^{max}}$. $x_{sc}^{max} = \sqrt{\frac{2\epsilon_{si}2\phi_b}{qN_a}} = 0.3\mu\text{m}$. (Here we've taken $2\phi_b = 0.7\text{V}$.) So: $C_{sc}(\phi_s = 2\phi_b) = 3.33 \times 10^{-8} \text{Fd/cm}^2$. And $C_{min} =$

$3.03 \times 10^{-8} \text{Fd/cm}^2$. C_{min} occurs “a tad” to the the right (for an n-channel) of threshold.

IIIb3. $C_{FB} = \frac{C_{ox}C_{sc}^{FB}}{C_{ox}+C_{sc}^{FB}}$. $C_{sc}^{FB} = \frac{\epsilon_{si}}{L_d}$. $L_d = \sqrt{\frac{\epsilon_{si}}{qN_a} \frac{kT}{q}} = 4.02 \times 10^{-6} \text{cm}$. $C_{sc}^{FB} = 2.5 \times 10^{-7} \text{Fd/cm}^2$. $C_{FB} = 1.45 \times 10^{-7}$. Flat band occurs at zero volts for insulators without defects and zero work-function difference materials.

IIIc. From the definitions of work function given on the board during the exam, the work function difference causes electrons to move from the field plate to the semiconductor. For a p-substrate, this enhances the depletion and you don’t have to put as much bias on the field plate to get it to invert. Thus, the whole CV plot is shifted left by 0.2V. For an n-type substrate, the work-function difference *accumulates* the surface, meaning you have to supply a more negative bias to create the positive inversion. Once again, the CV plot is just parallel shifted to the left by 0.2V.

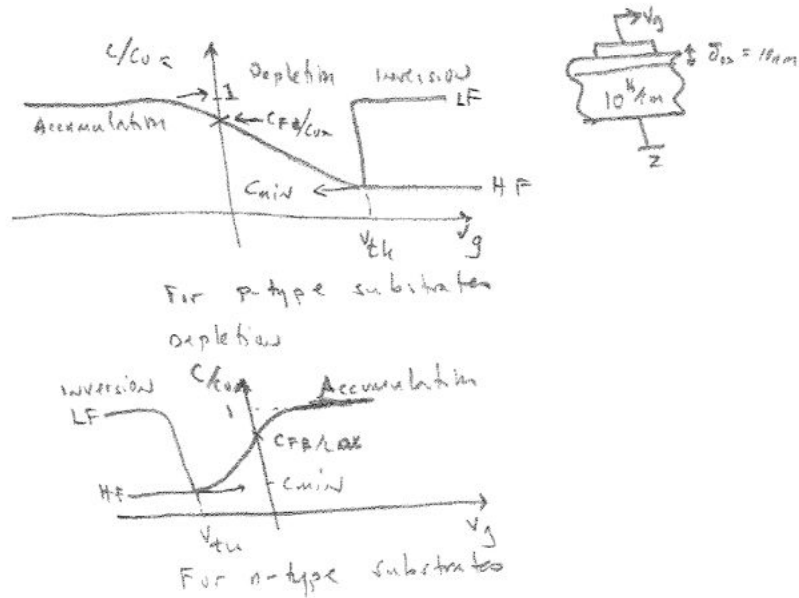


Figure 1: Capacitance voltage plots for n and p type substrates

IV1. We take the source/drain n+ plugs to be metallic and the “built-in” potential of these junctions is 0.7V. Thus, on the grounded source side the space-charge extent is:

$$x_{sc}^{source} = \sqrt{\frac{2\epsilon_{si}V_{bi}}{qN_a}} \quad (5)$$

On the drain side the space charge extent is:

$$x_{sc}^{drain} = \sqrt{\frac{2\epsilon_{si}(V_{bi} + |V_{ds}|)}{qN_a}} \quad (6)$$

At punch-through:

$$x_{sc}^{tot} = x_{sc}^{source} + x_{sc}^{drain} = \sqrt{\frac{2\epsilon_{si}V_{bi}}{qN_a}} + \sqrt{\frac{2\epsilon_{si}(V_{bi} + |V_{ds}|)}{qN_a}} = 0.13 \times 10^{-4} \quad (7)$$

or

$$\frac{1}{\sqrt{qN_a}}(\sqrt{2\epsilon_{si}V_{bi}} + \sqrt{2\epsilon_{si}(V_{bi} + |V_{ds}|)}) = 0.13 \times 10^{-4} \quad (8)$$

Given that The maximum drain bias is 3V and the built-in voltage is 0.7V, we can solve this equation for N_a . The result is: $5.64 \times 10^{17}/\text{cm}^2$. Or course, you may consider doping the substrate a little more to provide a safety margin. But this doping is pretty high as it is.

IV2. Lets be precise here (just for a change). That is, we won't use the $\phi_b = 0.7\text{V}$ approximation. (I won't penalize you if you did).

$$2\phi_b = 2\frac{kT}{q} \log_e\left(\frac{N_a}{n_i}\right) = 0.9\text{V} \quad (9)$$

and

$$V_{th} = 2\phi_b + \frac{\sqrt{2\epsilon_{si}qN_a2\phi_b}}{C_{ox}} \quad (10)$$

We use the value of C_{ox} given above for the 10nm oxide: $3.45 \times 10^{-7}\text{F}/\text{cm}^2$. This yields:

$$V_{th} = 0.9 + \frac{\sqrt{2 \times 10^{-12} \times 1.6 \times 10^{-19} \times 5.64 \times 10^{17} \times 0.9}}{3.45 \times 10^{-7}} = 2.07\text{V} \quad (11)$$

This is pretty high and it illustrates the problems and trade-offs of deep scaling! Can you see how to alleviate this high-threshold problem? How about varying C_{ox} ? But the high build-in voltage keeps the threshold above 1.