SOLUTIONS: ENEE 601 FINAL EXAM SPRING 2005

1a. This was basically just warm-up:

$$g_m = \frac{I_c}{V_{th}} = \frac{10^{-3}}{.0259} = 0.039\Omega^{-1} \tag{1}$$

$$r_\pi = \frac{\beta}{g_m} = 100/.039 = 2590\Omega \tag{2}$$

$$C_{diff} = \frac{I_c\tau}{V_{th}} = \frac{0.00110^{-4}}{.0259} = 3.86\mu F \tag{3}$$

b. This is a bit of an arithmetic slog. First, remember:

$$x_{sc} = \sqrt{\frac{2\epsilon_{si}(N_a + N_d)(\phi_{bi} - V)}{qN_aN_d}} \tag{4}$$

where:

$$\phi_{bi} = \frac{kT}{q} Ln\left[\frac{N_aN_d}{n_i^2}\right] \tag{5}$$

For the EB-Junction, in forward bias, the built-in voltage is .93V (a bit higher than the 0.7V we normally quote, but not that far off!); and the unbiased space charge thickness is: $0.1\mu m$. As we said in class, the rule of thumb is that in the forward biased diode, the space charge shrinks to half the "equilibrium" value: $0.05\mu m$. As the emitter is very heavily doped, you can take the space charge as all on the n-side. On the collector side (the CB junction), the built-in voltage is 0.75V and the space charge thickness is $0.81\mu m$. Note that in the forward active transistor the forward biased emitter potential sits at its built-in voltage and the drop across the diode is the collector supply rail minus that built in voltage. In this case, most of the space charge is in the lower-doped collector, and:

$$x_{sc,p} = \frac{N_a}{N_a + N_d} x_{sc} = \frac{1}{11} 0.81 = 0.074\mu m \tag{6}$$

Thus, the total space charge encroachment into the neutral base is $0.05\mu m + 0.081\mu m = 0.13\mu m$. The metallurgical junctions must be spaced *at least* this far apart to avoid punch through. There should be some safety margin as well. but that would be a processing issue beyond the scope of this course!

c. There are two sources of leakage in the collector-base diode: the diffusion currents induced by the supression of minority charge at the space charge boundaries, and space-charge generation current. The former is the saturation current, as high as $10^{-15}A$, as given in the board notes, and the latter is gotten from the Shockley-Reed formula:

$$I = qUV = q\frac{1}{\tau}\frac{(n_i^2 - np)}{(n + p + 2n_i)}Ax_{sc} \tag{7}$$

As the mobile densities are about zero in the space charge, this reduces to:

$$I = qUV = q\frac{1}{\tau}\frac{n_i}{n + p + 2n_i}Ax_{sc} = 1.6\times10^{-19}\times\frac{1.45\times10^{10}}{2\times10^{-4}}\times10^{-7}\times0.81\times10^{-4} = 0.93\times10^{-16}A \tag{8}$$

: note, this is about an order of magnitude lower than the diffusion current. BUT, in low-doped diodes (like PIN diodes, used in radiation detection) the space charge thickness is much larger, and the generation current can start to dominate.

So:

$$\frac{I_{noise}}{\sqrt{Hz}}(forward - shot) = \sqrt{2\times1.6\times10^{-19}\times10^{-3}} = 1.8\times10^{-11}A \tag{9}$$

$$\frac{I_{noise}}{\sqrt{Hz}}(reverse - diffusion) = \sqrt{2\times1.6\times10^{-19}\times10^{-15}} = 1.8\times10^{-16}A \tag{10}$$

$$\frac{I_{noise}}{\sqrt{Hz}}(space - charge - generation) = \sqrt{2\times1.6\times10^{-19}\times10^{-3}} = 5.5\times10^{-17}A \tag{11}$$

2.a The Fermi level of the p-type silicon is:

$$\phi_{fp} = 4.05 + .55 + \frac{kT}{q}Log\left[\frac{N_a}{n_i}\right] = 4.6 + 0.35 = 4.95Ev \tag{12}$$

This is 4.95 - 4.5 = .45 eV deeper than the metal Fermi Level. Thus, electrons will stream from the metal into the semiconductor, enhancing the space charge thickness, lowering the threshold by 0.45 V.

b. Only the occupied interface states contribute to flat-band shift. These states are donor-like. Thus, they must be above the Fermi-level to be occupied. For the p-type material, bands bend down and the whole lower half of the interface gap is unoccupied, as is a portion of the gap which is $1 - \phi_{bp}$ above that. For 10^{16}-doped material, $1 - \phi_{bp}$ is 0.35V. Thus, only a stretch of 0.2eV in the gap can be occupied, and:

$$\Delta V_{fb} = \frac{0.2\times1.6\times10^{-19}\times10^{11}}{C_{ox}} = \frac{0.32\times10^{-8}}{3.5\times10^{-7}} = 9mV \tag{13}$$

This is a small but sensible shift. The donor states are positively charged. Positive interface charge causes negative threshold shift.

3a. First, find the expected fluctuation in the dopant number under the gate:

$$\Delta N = \sqrt{N_a V} = \sqrt{10^{16} \times .1 \times 10^{-4} \times .03 \times 10^{-4} \times x_{sc}} \tag{14}$$

and, from eq. 4, we get x_{sc}, taking the bias to be a built in bias $= 2 \times 0.35 = 0.7V$. This yields $x_{sc} = 0.3\mu m$. Thus, the number of dopant atoms in the box is 9! and ΔN is 3. Thus, one can expect 33% fluctuations in doping density. Plugging this into the "intrinsic" threshold formula:

$$V_{th} = 2\phi_{bp} + \frac{\sqrt{2q\epsilon_{si}(N_a + 0.33N_a)2\phi_{bp}}}{C_{ox}} \tag{15}$$

gives the "high-end" threshold, and

$$V_{th} = 2\phi_{bp} + \frac{\sqrt{2q\epsilon_{si}(N_a - 0.33N_a)2\phi_{bp}}}{C_{ox}} \tag{16}$$

gives the low-end threshold. Thus, the "2σ" spread is: 0.86 - 0.81V = 50 mV. Getting significant!

b. Pinch-off occurs when the potential drop between the gate and the drain is just slightly less than one turn-on voltage. Thus, the inversion on the drain side must disappear - leaving only the fixed space-charge in the region. Thus, the drain bias can no longer exert direct control over the channel electric field. Of course, as the drain bias increases, the "pinch-off point" moves toward the source, and the effective channel length shrinks (giving rise to an Early-like behavior). But this increases the channel field as the square-root of drain bias (not like the linear effect you see in linear-triode. Thus, the I_{ds} curves flatten out as a function of drain voltage.

c. The drop across the pinched-off space charge is $V_{ds} - V_p$, where V_p is $V_{gs} - V_t$, where V_t is the threshold voltage. Thus, the physical extent of the pinched-off threshold is:

$$x_{sc} = \sqrt{\frac{2\epsilon_{si}(V_{ds} - V_p)}{qN_a}} = \sqrt{\frac{2\epsilon_{si}(V_{ds} - V_{gs} - V_t)}{qN_a}} \tag{17}$$

The amount of charge-per-unit-area in this region is $qN_a x_{sc}$. We can assume the drain is so heavily doped that it looks like a metal plug. Thus, this total sheet-charge is imaged in the drain diffusion, and :

$$E_{drain} = \frac{\sigma}{\epsilon_{si}} \tag{18}$$

or:

$$E_{drain} = \frac{qN_a x_{sc}}{\epsilon_{si}} = \frac{qN_a\sqrt{\frac{2\epsilon_{si}(V_{ds}-V_{gs}-V_t)}{qN_a}}}{\epsilon_{si}} = \frac{1}{\epsilon_{si}}\sqrt{2qN_a\epsilon_{si}(V_{ds}-V_{gs}-V_t)} \qquad (19)$$