2.6. (a) Using the integral expressions of Q_d and Q_i, one has

$$C_d \equiv -\frac{dQ_d}{d\psi_s} = \frac{qN_a(1-e^{-q\psi_s/kT})}{\mathscr{E}_s},$$

and

$$C_i \equiv -\frac{dQ_i}{d\psi_s} = \frac{q(n_i^2/N_a)(e^{q\psi_s/kT}-1)}{\mathscr{E}_s}.$$

Here $\mathscr{E}_s = \mathscr{E}(\psi = \psi_s)$.

(b) Take the square of Eq. (2.154) and differentiate with respect to ψ_s, one obtains

$$2Q_s\frac{dQ_s}{d\psi_s} = 2\varepsilon_{si}qN_a\left[\left(1-e^{-q\psi_s/kT}\right)+\frac{n_i^2}{N_a^2}\left(e^{q\psi_s/kT}-1\right)\right].$$

Using $Q_s = -\varepsilon_{si}\mathscr{E}_s$ (Gauss's law) and the above two equations in (a), it is straightforward to show that

$$C_{si} \equiv -\frac{dQ_s}{d\psi_s} = \frac{qN_a}{\mathscr{E}_s}\left[\left(1-e^{-q\psi_s/kT}\right)+\frac{n_i^2}{N_a^2}\left(e^{q\psi_s/kT}-1\right)\right] = C_d + C_i.$$

(c) When $\psi_s = 2\psi_B$, $\exp(q\psi_s/kT) = (N_a/n_i)^2 >> 1$ and $\exp(-q\psi_s/kT) << 1$. From (a), one has

$$C_i \cong C_d \cong \frac{qN_a}{\mathscr{E}_s}.$$

(d) It is clear from Fig. 2.25 that Q_s and therefore $\mathscr{E}_s$ takes off rapidly beyond strong inversion. This means that $C_d \propto \mathscr{E}_s^{-1}$ decreases rapidly beyond strong inversion. We say that the depletion layer (charge) is "screened" by the inversion layer. Note that C_i, on the other hand, increases rapidly beyond strong inversion because of the $\exp(q\psi_s/kT)$ factor.

2.8. Substituting Eq. (2.161) into Eq. (2.167), one obtains

$$V_g = \frac{\sqrt{2\varepsilon_{si} q N_a \psi_s}}{C_{ox}} + \psi_s,$$

which is basically a quadratic equation that can be solved for ψ_s:

$$\psi_s = V_g + \frac{\varepsilon_{si} q N_a}{C_{ox}^{\ 2}} - \sqrt{2\frac{\varepsilon_{si} q N_a}{C_{ox}^{\ 2}} V_g + \left(\frac{\varepsilon_{si} q N_a}{C_{ox}^{\ 2}}\right)^2}.$$

For differential changes in ψ_s and V_g, one goes back to the previous equation and obtains

$$\Delta V_g / \Delta \psi_s = 1 + \frac{\sqrt{\varepsilon_{si} q N_a / 2\psi_s}}{C_{ox}}.$$

This is simply $\Delta\psi_s = \Delta V_g/(1 + C_d/C_{ox})$, where C_d is the depletion charge capacitance given by Eq. (2.174).

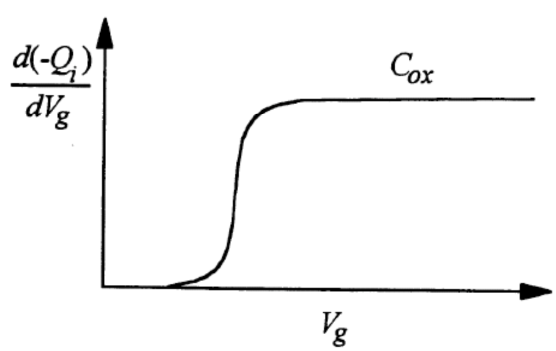
2.10. In Fig. 2.28(b), the total capacitance seen by the gate equals C_{ox} and $(C_i + C_d)$ connected in series, i.e.,

$$C \equiv \frac{d(-Q_s)}{dV_g} = \frac{C_{ox}(C_i + C_d)}{C_{ox} + C_i + C_d}.$$

Since $Q_s = Q_i + Q_d$ and $\Delta Q_i/\Delta Q_d = C_i/C_d$, it follows that

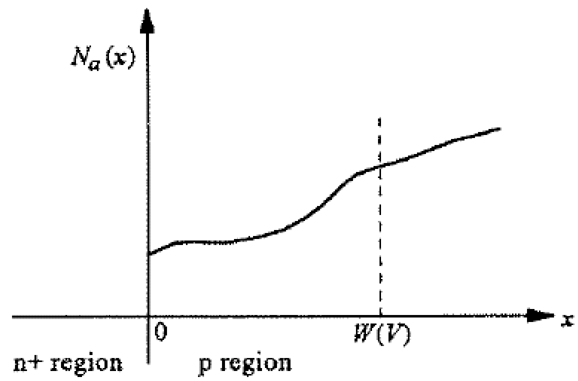
$$\frac{d(-Q_i)}{dV_g} = \frac{C_{ox} C_i}{C_{ox} + C_i + C_d}.$$

When V_g is below the threshold of strong inversion, $C_i << C_{ox}$, therefore $d(-Q_i)/dV_g \approx C_i/(1 + C_d/C_{ox}) << C_{ox}$. Above strong inversion, $C_i >> C_{ox}$ ($> C_d$) (see Exercise 2.6), and one has $d(-Q_i)/dV_g \approx C_{ox}$. Such a behavior is sketched schematically below.



The sharp transition takes place at the threshold voltage. $Q_i(V_g)$ is simply the integrated area under the above curve. Beyond threshold, Q_i increases linearly with V_g with a slope equal to C_{ox} (see Fig. 3.15 in the text).

2.14.



For the one-sided n$^+$-p diode, the incremental increase in the **magnitude** of the d layer charge dQ_d on the p-side as a result of an incremental increase in the applied voltage

$$dQ_d = qN_a(W)dW\,, \tag{1}$$

where the depletion-layer width W is a function of V. The depletion-layer capacitance is

$$C = \frac{dQ_d}{dV} = qN_a(W)\frac{dW}{dV}\,, \tag{2}$$

which gives

$$N_a(W) = \frac{C}{q(dW/dV)}. \tag{3}$$

From $$C = \frac{\varepsilon_{si}}{W}\,,$$

we have $$\frac{dW}{dV} = -\frac{\varepsilon_{si}}{C^2}\frac{dC}{dV} = \frac{\varepsilon_{si}C}{2}\frac{d(1/C^2)}{dV}. \tag{4}$$

Substituting (4) into (3), we have

$$N_a(W) = \frac{2}{q\varepsilon_{si}[d(1/C^2)/dV]}.$$

2.15. Referring to Fig. 2.14 and Eq. (2.75), we have

$$\psi_m = \psi_{bi} + V = \frac{\mathscr{E}_m(W_d + d)}{2}\,,$$

where V is the applied voltage. From Gauss's law we have

$$Q_d = \varepsilon_{si}\mathscr{E}_m . \tag{2}$$

Therefore,

$$\frac{1}{C_d} = \frac{dV}{dQ_d} = \frac{d\left[\mathscr{E}_m (W_d + d)/2\right]}{d\left[\varepsilon_{si}\mathscr{E}_m\right]} = \frac{1}{2\varepsilon_{si}}\left[(W_d + d) + \mathscr{E}_m \frac{dW_d}{d\mathscr{E}_m}\right]. \tag{3}$$

Now, from Eq. (2.74), we have

$$\mathscr{E}_m \frac{dW_d}{d\mathscr{E}_m} = \mathscr{E}_m \frac{d(x_n + x_p)}{d\mathscr{E}_m} = \mathscr{E}_m \frac{dx_n}{d\mathscr{E}_m} + \mathscr{E}_m \frac{dx_p}{d\mathscr{E}_m} = (x_n - d) + x_p . \tag{4}$$

Substituting (4) into (3), we have

$$\frac{1}{C_d} = \frac{W_d}{\varepsilon_{si}} .$$

2.17. (a) Ignoring heavy doping effect, from Eq. (2.110) the electron saturation current density is

$$J_n = \frac{qD_n n_i^2}{N_a L_n} = \frac{qn_i^2}{N_a}\frac{L_n}{\tau_n}, \tag{1}$$

where we have used the fact that $(D/L) = (L/\tau)$. From Table 2.1, we have $n_i = 1.4\times10^{10}$ cm^{-3}, and from Fig. 2.18, for $N_a = 10^{17}$ cm^{-3}, we have $L_n = 75$ μm $= 7.5\times10^{-3}$ cm, and $\tau_n = 3\times10^{-6}$ s. Substituting these values into (1) gives $J_n = 7.8\times10^{-13}$ A/cm^2. Similarly, the hole saturation current density is

$$J_p = \frac{qD_p n_i^2}{N_d L_p} = \frac{qn_i^2}{N_d}\frac{L_p}{\tau_p}. \tag{2}$$

From Fig. 2.18, for $N_d = 10^{20}$ cm^{-3}, we have $L_p = 0.38$ μm $= 3.8\times10^{-5}$ cm, and $\tau_p = 5\times10^{-10}$ s. Substituting these values into (2) gives $J_p = 2.4\times10^{-14}$ A/cm^2.

(b) If the n$^+$-region is shallow, with a width of $W_n = 0.1$ μm $= 1\times10^{-5}$ cm, the hole saturation current density is

$$J_p = \frac{qD_p n_i^2}{N_d W_n} = \frac{qn_i^2}{N_d}\frac{L_p^2}{\tau_p W_n}. \tag{3}$$

Using the same values as in (a), we have $J_p = 9.1\times10^{-14}$ A/cm^2.

(c) If heavy-doping effect is included in (b), then the hole saturation current density is given by

$$J_p = \frac{qD_p n_{ie}^2}{N_d W_n} = \frac{qn_i^2}{N_d}\frac{L_p^2}{\tau_p W_n}\exp(\Delta E_g / kT). \tag{4}$$

From Fig. 6.3, for an n$^+$-region doped to 10^{20} cm^{-3}, $\Delta E_g \approx 92$ meV. At room temperature (300 K), $kT = 26$ meV, and $\exp(\Delta E_g/kT) = 34$. That is, the effect of heavy doping increases J_p by a factor of 34, giving $J_p = 3.1\times10^{-12}$ A/cm^2.