2.18. Consider a wide-emitter but narrow-base n$^+$-p diode, with emitter doping concentration $N_E = 10^{20}$ cm^{-3}, base doping concentration $N_B = 10^{17}$ cm^{-3}, and base width $W_B = 100$ nm.

(a) Ignoring heavy-doping effect, Eq. (2.143) gives the diffusion capacitance ratio

$$\frac{C_{Dn}}{C_{Dp}} = \frac{N_E}{N_B}\frac{W_B}{2L_{pE}} . \qquad (1)$$

and

$$n_{ieE}^2 = n_i^2 \exp(\Delta E_{gE} / kT) .$$

Therefore,

$$\frac{n_{ieB}^2}{n_{ieE}^2} = \exp\left(\frac{\Delta E_{gB} - \Delta E_{gE}}{kT}\right) . \qquad (8)$$

From Fig. 6.3, we have $\Delta E_{gB} \approx 2$ meV for p-type base with doping concentration of 10^{17} cm^{-3}, and $\Delta E_{gE} \approx 92$ meV for n-type emitter with doping concentration of 10^{20} cm^{-3}. Also, at room temperature (300 K), $kT = 26$ meV. Therefore

$$\frac{n_{ieB}^2}{n_{ieE}^2} = \exp\left(\frac{\Delta E_{gB} - \Delta E_{gE}}{kT}\right) = 0.031. \qquad (9)$$

3.1. At low drain biases or near the source, $V = 0$. Given V_g, ψ_s can be solved from Eq. (3.14). Since $q\psi_s/kT$ is small compared with $(n_i^2/N_a^2)\exp(q\psi_s/kT)$ beyond strong inversion, one has

$$V_g = V_{fb} + \psi_s + \frac{\sqrt{2\varepsilon_{si} kTN_a}}{C_{ox}}\left(\frac{n_i}{N_a}\right)e^{q\psi_s/2kT}.$$

When V_g increases beyond the point where $\psi_s = 2\psi_B$, most of that increase appears as inversion charge in the third term. There is very little change in the band bending beyond $\psi_s = 2\psi_B$. A good approximate solution for ψ_s is then obtained by letting $\psi_s = 2\psi_B$ for the second term and solving ψ_s from the third term:

$$\psi_s \approx 2\psi_B + \frac{2kT}{q}\ln\left(\frac{C_{ox}(V_g - V_{fb} - 2\psi_B)}{\sqrt{2\varepsilon_{si} kTN_a}}\right).$$

Here $\exp(q\psi_B/kT) = N_a/n_i$ has been applied.

It is clear from the above equation that $\psi_s - 2\psi_B$ is a weak function of V_g. As an example, for $N_a = 10^{16}$ cm^{-3}, $t_{ox} = 200$ Å, and $V_g - V_{fb} = 5$ V, one has $2\psi_B = 0.7$ V, and $\psi_s - 2\psi_B \approx 8kT/q \approx$ 0.2 V.

3.2. Eq. (3.14) is an implicit equation relating ψ_s and V. Taking a differential yields

$$0 = d\psi_s + \frac{\sqrt{2\varepsilon_{si} kTN_a}}{2C_{ox}}\left[\frac{q\psi_s}{kT} + \frac{n_i^2}{N_a^2}e^{q(\psi_s - V)/kT}\right]^{-1/2}\frac{q}{kT}\left(d\psi_s + \frac{n_i^2}{N_a^2}e^{q(\psi_s - V)/kT}(d\psi_s - dV)\right).$$

It is straightforward to re-group the terms and show that

$$\frac{d\psi_s}{dV} = \frac{(n_i^2/N_a^2)e^{q\psi_s/kT}}{1 + (n_i^2/N_a^2)e^{q\psi_s/kT} + (C_{ox}^2/\varepsilon_{si} qN_a)(|Q_s|/C_{ox})}$$

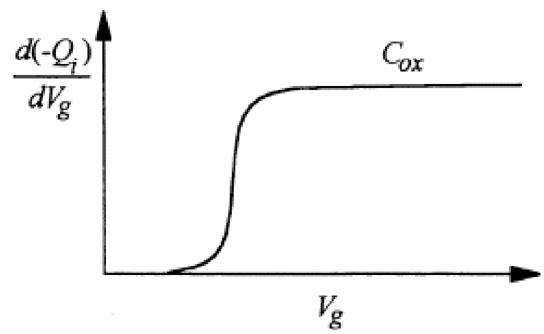
in the limit of $V \to 0$, where

$$|Q_s| = \sqrt{2\varepsilon_s kTN_a}\left[\frac{q\psi_s}{kT} + \frac{n_i^2}{N_a^2}e^{q\psi_s/kT}\right]^{1/2}.$$

3.4. This exercise is closely related to Exercise 2.10, in which Eq. (3.57) is derived:

$$\frac{d(-Q_i)}{dV_g} = \frac{C_{ox}C_i}{C_{ox} + C_i + C_d}.$$

When V_g is below the threshold of strong inversion, $d(-Q_i)/dV_g \approx C_i/(1 + C_d/C_{ox}) << C_{ox}$. Above strong inversion, $C_i >> C_{ox}$ ($> C_d$), and one has $d(-Q_i)/dV_g \approx C_{ox}$. This behavior is sketched below.



$Q_i(V_g)$ is simply the integrated area under the curve. Beyond threshold, Q_i increases linearly with V_g with a slope equal to C_{ox} (see Fig. 3.15 in the text).

The sharp transition above can be used to define a kind of *inversion charge threshold voltage*, V_t^{inv}, where $C_i = C_{ox}$. Since $C_d \rightarrow 0$ due to screening by inversion charge (Exercise 2.6), $d|Q_i|/dV_g \approx C_{ox}C_i/(C_{ox}+C_i) = C_{ox}/2$ at $V_g = V_t^{inv}$. Also, from Eq. (2.178), one has $Q_i \approx (2kT/q)C_{ox}$ at this point. Such a threshold voltage is slightly higher than the "$2\psi_B$" threshold where $C_i = C_d$.

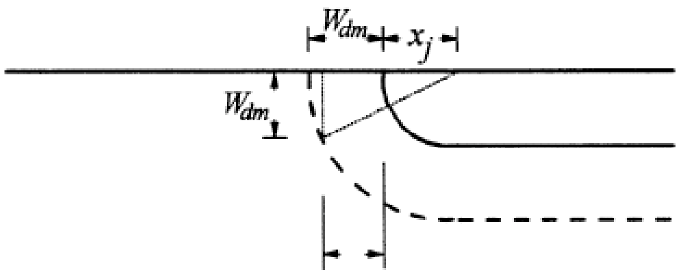
3.5. From the last term of Eq. (3.59), the polysilicon depletion effect causes an inversion charge loss of

$$\Delta Q_i = \frac{C_{ox}^{\ 3}(V_g - V_t)^2}{2\varepsilon_{si}qN_p}.$$

If we neglect the depletion charge in bulk silicon, then $Q_p \approx Q_i \approx C_{ox}(V_g - V_t)$ and the above equation can be written as $\Delta Q_i/Q_i \approx C_{ox}Q_p/2\varepsilon_{si}qN_p$. From the discussions below Eq. (2.184), the polysilicon depletion capacitance is $C_p = \varepsilon_{si}qN_p/Q_p$. Therefore, one has $\Delta Q_i/Q_i \approx C_{ox}/2C_p$.

There is a factor of two difference between the loss of charge and the loss of capacitance because charge is the integration of capacitance and the polysilicon depletion capacitance is voltage dependent (the next increment of charge always appears at the far edge of the poly depletion region). This shows that treating the polysilicon depletion region as an equivalent oxide layer will over estimate its effect on the drain current.

3.6. Following basic trigonometry in the diagram below,



one has

$$\frac{L-L'}{2} = \sqrt{(x_j + W_{dm})^2 - W_{dm}^{\ 2}} - x_j = x_j\left(\sqrt{1 + 2W_{dm} / x_j} - 1\right).$$

From which it is easy to show that

$$\frac{L+L'}{2L} = 1 - \frac{x_j}{L}\left(\sqrt{1 + \frac{2W_{dm}}{x_j}} - 1\right).$$

In Fig. 3.19, the depletion charge in a short-channel device is proportional to the area of the trapezoid region, i.e., $Q_B' = qN_aWW_{dm}(L + L')/2$, which is less than that of the long channel device, $Q_B = qN_aWW_{dm}L$. From Eq. (3.62), $\Delta V_t(\text{SCE}) = (Q_B - Q_B')/WLC_{ox}$; therefore,

$$\Delta V_t(\text{SCE}) = \frac{qN_aW_{dm}}{C_{ox}}\left(\sqrt{1 + \frac{2W_{dm}}{x_j}} - 1\right)\frac{x_j}{L}.$$