Solutions to ENEE 601MidTerm Exam - Spring 2006

1.a First, we calculate the "intrinsic " resistivity using Matthiessien's rule for addition or resistances due to different transport processes:

$$\frac{1}{\rho_{total}} = \frac{1}{\rho_{holes}} + \frac{1}{\rho_{electrons}} \tag{1}$$

$$\rho_{total} = en_i\mu_{hole} + en_i\mu_{electron} \tag{2}$$

Note: in intrinsic silicon the number of electrons equals the number of holes (n_i). Thus, equation 2 solves as ρ_{total} = 215,517 Ω - cm.

Now let's see what happens when we add hole (acceptor) dopant. Again, we use Matthiessien's rule for addition or resistances due to different transport processes. We also solve for the new electron concentration using the law of mass action (consistency of the pn-product):

$$\frac{1}{\rho_{total}} = e(1.73n_i)\mu_{hole} + e\frac{n_i^2}{1.73n_i}\mu_{electron} \tag{3}$$

which becomes, (for $N_a = 1.73n_i$) ρ_{total} = 248,858 Ω - cm.

1.b Why does the resistivity increase on doping? I was interested in a coherent explanation of the above calculations. Such an explanation proceeds as follows. The consistency of the pn-product tells us that when we add more holes, we force electrons out of the conduction band. Thus we are replacing a high mobility carrier by a low mobility carrier. This increases resistivity, even though the number density of mobile carriers may not change. If you included scattering as a cause, I took half off. Man, am I mean!

NOTE: This has **nothing** to do with scattering. The hole has lower effective mass than the electron (because it's dispersion curve, the E-K relationship, differs from that of the electron. The calculation in (a) didn't include scattering as a factor in any event. Also, in the limit of low doping (surely less than 10^{14} the phonon defined scattering lengths are so much shorter than the impurity defined scattering lengths that small changes in doping have no effect on scattering. Impurity scattering doesn't set in until you get above 10^{15}.

2a. Since the doping is equal (but opposite in sign) on each side of the junction, the total space charge extent is:

$$x_{sc} = \sqrt{\frac{4\epsilon_{si}(\phi_{bi} - V_a)}{qN}} \tag{4}$$

We can use the log formula to get the built-in voltage; or, we can be really lazy and just claim the built-in voltage is always around 0.7V. I will opt for the latter. Now let's visualize the charge system. We have two "dipole" sheets of equal thickness ($x_{sc}/2$) pressed against one another. The charge density in the sheet (charge per unit area, or σ) is:

$$\sigma = \frac{1}{2}x_{sc}qN = \frac{1}{2}\sqrt{\frac{4\epsilon_{si}(\phi_{bi} - V_a)}{qN}}qN = \sqrt{\epsilon_{si}(\phi_{bi} - V_a)qN} \tag{5}$$

From the Gauss' Law construction, the electric field from each sheet is $\sigma/(2\epsilon_{si})$. As there are 2 sheet providing electric force in the same direction, the greatest electric field occurs right at the metallurgical junction and is equal to $\sigma/(\epsilon_{si})$. As we know that the maximum sustainable field is 10^5V/cm, We can write:

$$10^5 = \frac{1}{\epsilon_{si}}\sqrt{\epsilon_{si}(0.7 - V_a)qN} \tag{6}$$

and solve for Va (remembering that this is reverse bias, and V_a is negative) when N is 10^{17}. Our reference text, Muller and Kamins, derive this formula and neglect the built-in bias, ϕ_{bi}. They do this because usually the break down voltages are large compared to the built-in voltages *in most cases.*But they do acknowledge this is an approximation. If we neglect the built-in drop, this yields $V_a = 0.625$V to breakdown. Really tiny. Actually, the breakdown field is doping dependent. For the case at hand the breakdown field is actually 4 times the value listed, giving a breakdown voltage of 9.3 V (including the built-in field effect). The measured breakdown voltage of such a system (as given in Sze) is 10 V. Pretty close!

If you take the built-in field into account, the breakdown field is about zero. The appearance is that the internal space-charge field is strong enough to set the system into breakdown even without bias. Of course, this cannot happen. If the system started to breakdown, more mobile charge would be present than the abrupt junction model would predict and we'd have to re-slove the transport equation system for a new equilibrium. Very hard to do - not expected on an exam!

Many exam responses used the following (erroneous) reasoning. Since the space charge thickness is small, so can assume a linear drop in the depletion, and:

$$|E| = \frac{\partial V}{\partial x} = \frac{V_a}{x_{sc}} \tag{7}$$

As there is (in reality) a quadratic potential vs position variation in the space charge, this equation isn't true. I gave half credit for this approach - even though the resulting formula isn't correct. It's off by a built-in voltage in the numerator, necessitating use of the quadratic formula to solve. The reason I gave half credit is that our main text derives the maximum field by directly integrating the Poisson equation, achieving the same formula as shown in eq. 7. So the linear approximation yields a result which is *almost* right - interestingly enough.

3.a Donor states are positive when above the Fermi level and neutral when below. At threshold, the "band bending" (surface potential) is $2\phi_b = 0.7$V for the case at hand. Define the gap energy (in volts) as V_g. Thus, the Fermi level right at the surface is Vg/2 - 0.35 = .55 - .35 = 0.2V below the conduction band edge, and 0.9V above the valence band edge. For the donor states, only those above the Fermi level are occupied. The area density of **occupied** interface charge is: 0.25 q N_{it} = 0.25 $1.6\times10^{-19} \times 10^{11} = 0.4 \times10^{-8}$C/cm^2. We divide this by $C_{ox} = \epsilon_{ox}/d_{ox} = 3.45 \times 10^{-13}/10^{-6} = 3.45 \times 10^{-7}$Fd/cm^2 to get the threshold shift. This gives a 12 mV shift. As the states are charged *positively* the threshold is lowered (made more negative). So, it seems that really thin oxides don't suffer from interface state induced V_{th} shift as much as thick oxides.

4.a We take the active channel length as the separation of the source and drain metallurgical junctions (3μm). As the source is grounded, and the source is much more heavily doped than the substrate. We can take the drain junction space charge as fixed in extend, and assume that the space charge is only in the substrate side of the junction. I'd use the "lazy" approximation for the built-in voltage, but I'd jack it up about 0.1 V to account for the heavy source doping: $\phi_{bi} = 0.8$V. Thus,

$$x_{sc} = \sqrt{\frac{2\epsilon_{si}0.8}{1.6 \times 10^{-19}10^{16}}} = 0.32\mu m \tag{8}$$

Again, let us say that the drain space charge extends only into the channel. We can write for the reach-thru condition:

$$3 \times 10^{-4} = 0.32 \times 10^{-4} + x_{sc}^{drain} = 0.32 \times 10^{-4} + \sqrt{\frac{2\epsilon_{si}(0.8 - V_a)}{qN_{sub}}} \tag{9}$$

where N_{sub} is 10^{16}. We can solve this equation for Va, the reverse bias voltage needed to punch through the channel. Here, V_a solves as 56.7V.

4.b Without *any* calculation, we know that the drain space charge on the substrate side must be 3 - 0.32 = 2.68 microns across. This, the area charge density in the space charge is just $2.68 \times10^{-4} \times 1.6 \times 10^{-19} \times 10^{16} = 4.3 \times 10^{-7}$C/cm^2. If we multiply this by two (to account for the image charge in the drain doping) and divide by the dielectric permittivity of silicon, we get the maximum electric field (which, again, occurs at the metallurgical junction): 8.6×10^5V/cm. This is above the breakdown field of silicon. So the breakdown would probably be reached before punch-thru.

EXTRA CREDIT

As always, Q = CV. For the case at hand, the charge is Q_f, which is right near the oxide-semiconductor interface. Thus, it induces all its band-bending in the silicon. and the Q is just the full Q_f. C is $C_{ox} = 3.45 \times 10^{-13}/10^{-6} = 3.45 \times 10^{-7}$Fd/cm^2. For a 1mV observable shift, the minimum observed Q_f is 3.45×10^{-7}Fd/cm$^2 \times 10^{-3}$V$= 3.45 \times 10^{-10}$C/cm^2. This converts to 2.16×10^{-9} charges per square centimeter.