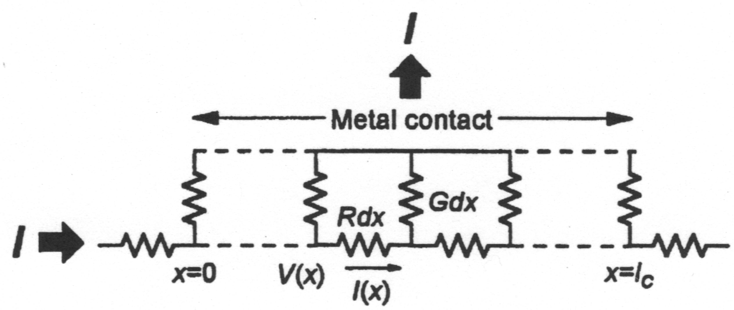
5.5. The *transmission line model* of contact resistance in a planar geometry is represented by the distributed network below. The current flows from a thin resistive film (diffusion with a sheet resistivity ρ_{sd}) into a ground plane (metal) with an interfacial contact resistivity ρ_c between them (Fig. 5.13).



Following a similar approach as in Eqs. (5.20)-(5.22), one can write

$$V(x+dx)-V(x)=\frac{dV}{dx}dx=-I(x)Rdx,$$

and

$$I(x+dx)-I(x)=\frac{dI}{dx}dx=-V(x)Gdx.$$

Here $R=\rho_{sd}/W$ and $G=W/\rho_c$. From the above two equations, one obtains

$$\frac{d^2f}{dx^2}=RGf=\frac{\rho_{sd}}{\rho_c}f,$$

where $f(x)=V(x)$ or $I(x)$.

5.7. From Eqs. (5.35) and (5.37), $\tau_{bmin}<\tau$ if

$$C_{out}+C_L>2C_{out}+2\sqrt{C_{in}C_L}.$$

This inequality is quadratic in $C_L^{1/2}$, which can be solved to yield

$$C_L>\left(\sqrt{C_{in}}+\sqrt{C_{in}+C_{out}}\right)^2.$$

Under this condition, the insertion of one or a few properly designed buffer stage(s) will help reduce the overall delay.

6.1. Consider hole current flow in an n-region. Eq. (2.46) gives

$$J_p = -qp_n\mu_p \frac{d\phi_p}{dx}. \tag{1}$$

Now, as shown in section 2.2.3.1, $d\phi_n$(n-region)/$dx \approx 0$. Therefore, (1) can be rewritten as

$$J_p(n-region) \approx -qp_n\mu_p \frac{d}{dx}(\phi_p - \phi_n). \tag{2}$$

Eq. (6.16) gives

$$(\phi_p - \phi_n) = \frac{kT}{q}\ln\left(\frac{p_n n_n}{n_{ie}^2}\right) \tag{3}$$

for the n-region. Substituting (3) into (2), we obtain

$$J_p(n-region) = qp_n\mu_p\left[-\frac{kT}{q}\left(\frac{1}{n_n}\frac{dn_n}{dx} - \frac{1}{n_{ie}^2}\frac{dn_{ie}^2}{dx}\right)\right] - qD_p\frac{dp_n}{dx}, \tag{4}$$

where we have used the Einstein relation $D_p = kT\mu_p/q$. If we compare (4) with Eq. (2.44), i.e.,

$$J_p(n-region) = qp_n\mu_p\mathscr{E}(n-region) - qD_p\frac{dp_n}{dx}, \tag{5}$$

we have

$$\mathscr{E}(n-region) = \left[-\frac{kT}{q}\left(\frac{1}{n_n}\frac{dn_n}{dx} - \frac{1}{n_{ie}^2}\frac{dn_{ie}^2}{dx}\right)\right].$$

6.4. Eq. (6.139) gives

$$\beta(\omega) = \left[\frac{1}{\beta_0} + j\omega\left(\tau_F + \frac{(C_{dBE} + C_{dBC})}{g_m} + C_{dBC}(R_L + r_e + r_c)\right)\right]^{-1}. \quad (1)$$

In the high-frequency limit, and for short circuit ($R_L = 0$), (1) becomes

$$\beta(\omega) \approx \left[j\omega\left(\tau_F + \frac{(C_{dBE} + C_{dBC})}{g_m} + C_{dBC}(r_e + r_c)\right)\right]^{-1}. \quad (2)$$

The magnitude of $\beta(\omega)$ becomes unity at $\omega = \omega_T = 2\pi f_T$, or

$$1 \approx 2\pi f_T\left(\tau_F + \frac{(C_{dBE} + C_{dBC})}{g_m} + C_{dBC}(r_e + r_c)\right). \quad (3)$$

Eq. (6.99) gives

$$g_m = \frac{qI_C}{kT}. \quad (4)$$

Therefore, (3) gives

$$\frac{1}{2\pi f_T} = \tau_F + \frac{kT}{qI_C}(C_{dBE} + C_{dBC}) + C_{dBC}(r_e + r_c).$$

6.5. Neglecting parasitic resistances, Eqs. (6.152) and (6.153) give

$$I_E = -I_{EBO}\left[\exp(qV'_{BE}/kT)-1\right] - \alpha_R I_C \quad (1)$$

and

$$I_C = -I_{CBO}\left[\exp(qV'_{BC}/kT)-1\right] - \alpha_F I_E, \quad (2)$$

which can be rearranged to give

$$V'_{BE} = \frac{kT}{q}\ln\left[1-\frac{I_E+\alpha_R I_C}{I_{EBO}}\right] \quad (3)$$

and

$$V'_{BC} = \frac{kT}{q}\ln\left[1-\frac{I_C+\alpha_F I_E}{I_{CBO}}\right]. \quad (4)$$

Now,

$$V'_{CE} = V'_C - V'_E = (V'_C - V'_B) + (V'_B - V'_E) = V'_{BE} - V'_{BC}. \quad (5)$$

Substituting (3) and (4) into (5) and rearranging, we obtain

$$V'_{CE} = \frac{kT}{q}\ln\left[\frac{I_{CBO}(I_{EBO}-I_E-\alpha_R I_C)}{I_{EBO}(I_{CBO}-I_C-\alpha_F I_E)}\right]. \quad (6)$$

Now, Eq. (6.90) gives

$$\alpha_R I_{R0} = \alpha_F I_{F0}, \quad (7)$$

and Eqs. (6.150) and (6.151) give

$$I_{EBO} \equiv I_{F0}(1-\alpha_R\alpha_F), \quad (8)$$

and

$$I_{CBO} \equiv I_{R0}(1-\alpha_R\alpha_F). \quad (9)$$

Therefore,

$$\frac{I_{CBO}}{I_{EBO}} = \frac{I_{R0}}{I_{F0}} = \frac{\alpha_F}{\alpha_R}. \quad (10)$$

Substituting (10) into (6) gives

$$V'_{CE} = \frac{kT}{q}\ln\left[\frac{\alpha_F(I_{EBO}-I_E-\alpha_R I_C)}{\alpha_R(I_{CBO}-I_C-\alpha_F I_E)}\right].$$