ENEE 621: ESTIMATION AND DETECTION THEORY

Midterm Examination II

Spring 2007

Narayan **Maximum 60 points**

Answer all *three* questions. Please be precise and rigorous in your answers. Show all your calculations.

Name:

Problem 1 [10 points]

A rv θ, with $P(\theta = -1) = P(\theta = 1) = \frac{1}{2}$, is to be estimated on the basis of an observed rv Y where

$$Y = \theta + N$$

with $N \sim \mathcal{N}(0,1)$ being independent of θ.

Determine the mean-error magnitude (MEM) estimate $g_{MEM}(y)$ of θ given $Y = y$, $-\infty < y < \infty$.

Problem 2 [25 points]

Consider the Bayesian estimation problem of estimating a rv θ on the basis of an observed rv Y, where

$$Y = \theta + V$$

with $\theta \sim unif([0,1])$ and independent of the rv V where

$$P(V \leq v) = 1 - e^{-\alpha v}, \ v \geq 0$$

for $\alpha > 0$ a known constant.

[**2a**] [**10 points**] Compute the conditional probability density function of θ given $Y = y$, $y \geq 0$.

[**2b**] [**10 points**] Determine the minimum mean-squared error (MSE) estimate $g_{MSE}(y)$ of θ given $Y = y$, $y \geq 0$.

[**2c**] [**5 points**] Determine the maximum a posteriori (MAP) estimate $g_{MAP}(y)$ of θ given $Y = y$, $y \geq 0$.

Problem 3 [25 points]

Recursive prediction of a moving average process

Consider a $\mathbb{R}$-valued "moving average" process $\{X_t\}_{t=0}^{\infty}$ defined by

$$X_t = W_t + W_{t-1}, \qquad t \geq 0$$

where $\{W_t\}_{t=-1}^{\infty}$ is a $\mathbb{R}$-valued zero-mean stationary white (i.e., uncorrelated) noise process with unit variance.

[3a] [10 points] Denoting by $\hat{X}_{t+1|t} = \hat{\mathrm{E}}[X_{t+1}|X^t] = \hat{\mathrm{E}}[X_{t+1}|X_0, X_1, \cdots, X_t]$ the linear mean-squared error estimate of X_{t+1} given $(X_0, X_1, \cdots, X_t)$, derive a recursion for $\hat{X}_{t+1|t}$ in terms of $\hat{X}_{t|t-1}$ and X_t, $t \geq 0$. Assume that $\hat{X}_{0|-1} = \mathrm{E}[X_0]$.

[3b] [15 points] Show that the error variance equals

$$\mathrm{E}\left[\left(X_t - \hat{X}_{t|t-1}\right)^2\right] = \frac{t+2}{t+1}.$$