

ENEE 621
FALL 2015
DETECTION AND ESTIMATION THEORY

TEST # 1:

Please work out the **four** (4) attached problems. **Show** work on provided space and **explain** reasoning; **box** or **circle** your final answers.

Please write your **full name** and **SID** in the space provided below! Thank you for your cooperation.

Problem #1 /30

Problem #2 /30

Problem #3 /30

Problem #4 /30

Total /120

NAME/SID: _____

1.

An \mathbb{R}_+ -valued rv Y is said to exponentially distributed with parameter $\alpha > 0$, written $Y \sim \text{Exp}(\alpha)$, if

$$\mathbb{P}[Y \leq y] = 1 - e^{-\alpha y^+}, \quad y \in \mathbb{R}$$

with $y^+ = \max(0, y)$. Consider now the simple binary hypothesis testing problem

$$H_1 : Y \sim \text{Exp}(\alpha_1)$$

$$H_0 : Y \sim \text{Exp}(\alpha_0)$$

with $0 < \alpha_0 < \alpha_1$.

1.a. For each $\eta \geq 0$, explicitly describe the test $d_\eta : \mathbb{R} \rightarrow \{0, 1\}$. In particular what is the set $C(d_\eta)$ where the null-hypothesis H_0 is accepted? (**6 pts.**).

1.b. For each $\eta \geq 0$, compute the probability $P_F(d_\eta)$ of false alarm and the probability $P_D(d_\eta)$ of detection under the test $d_\eta : \mathbb{R} \rightarrow \{0, 1\}$ (**8 pts.**).

1.c. Compute the Bayesian cost $V : [0, 1] \rightarrow \mathbb{R}$ when using the probability of error criterion (**10 pts.**).

1.d. Find the ROC curve in explicit form (**6 pts.**).

ANSWER:

2. _____

2.a.
(4 pts.) (6 pts.).

2.b. (6 pts.).

2.c.
(10 pts.).

ANSWER:

3. _____

3.a.

3.b.

(5 pts.).

3.c. (5 pts.).

ANSWER:

#4.

4.a.
(10 pts.).

4.b.
(10 pts.).

4.c.
(10 pts.).

ANSWER:

