ESTIMATION AND DETECTION THEORY

HOMEWORK # 1:

Please work out the **ten** (10) problems stated below – HVP refers to the text: H. Vincent Poor, *An Introduction to Signal Detection and Estimation* (Second Edition), Springer Texts in Electrical Engineering Springer, New York (NY), 2010. With this in mind, Exercise **II.2** (HVP) refers to Exercise 2 for Chapter II of HVP. Exercises are located at the end of each chapter.

Show work and **explain** reasoning.

1.

Solve Exercise **II.1** (HVP).

2.

Solve Part (a) of Exercise **II.2** (HVP).

3.

Solve Part (a) of Exercise **II.3** (HVP)

A definition

Let I denote an interval of $\mathbb{R}$, not necessarily finite, closed or open. A function $g : I \to \mathbb{R}$ is a *concave* function if for arbitrary x_0 and x_1 in I, it holds that

$$(1-\lambda)g(x_0) + \lambda g(x_1) \leq g((1-\lambda)x_0 + \lambda x_1) \tag{1.1}$$

for each λ in $[0, 1]$.

4.

Let I denote an interval of $\mathbb{R}$, not necessarily finite, closed or open, and let A be an arbitrary index set. For each α in A, let $f_\alpha : I \to \mathbb{R}$ be a concave function. With the function $g : I \to \mathbb{R}$ defined by

$$g(x) = \inf\left(f_\alpha(x) : \ \alpha \in A\right), \quad x \in I$$

show that the mapping $g : I \to \mathbb{R}$ is concave.

5.

Let I be an open interval, say (a, b) with $a < b$ in $\mathbb{R}$. Show that a concave mapping $g : I \to \mathbb{R}$ is necessarily continuous on I [**HINT:** Use the definition (1.1).]

6.

Let I be an interval which is *open*, say of the form $[a,b]$, $(a,b]$ or $[a,b)$ with $a < b$ in $\mathbb{R}$. Is it still true that a concave mapping $g : I \to \mathbb{R}$ is necessarily continuous on I? Either give a proof or exhibit a counterexample.

7.

Solve Part (a) of Exercise **II.4** (HVP).

8.

Consider the binary hypothesis testing problem

$$\begin{array}{ll} H_1 : & \boldsymbol{Y} \sim F_1 \\ H_0 : & \boldsymbol{Y} \sim F_0. \end{array}$$

where F_0 is a discrete distribution uniform on $\{0,1\}$, and F_1 is uniform on the interval $(0,1)$. Derive the test that minimizes the probability of error. Assume an arbitrary prior p in $(0,1)$.

9.

You are being told that an observation Y can be characterized as

$$\begin{array}{ll} H_1 : & \boldsymbol{Y} \sim Z^2 \\ H_0 : & \boldsymbol{Y} \sim e^Z \end{array}$$

where $Z \sim \mathrm{N}(m, \sigma^2)$. Can this situation be formulated as a binary hypothesis testing problem? Explain. In the affirmative specify F_0 and F_1.

10.

A rv Z is said to be Rayleigh distributed with parameter $\sigma^2 > 0$ if its probability distribution F_{σ^2} admits a probability density function $f_{\sigma^2} : \mathbb{R} \to \mathbb{R}_+$ given by

$$f_{\sigma^2}(z) = \begin{cases} 0 & \text{if } z < 0 \\ \\ \frac{z}{\sigma^2} e^{-\frac{z^2}{2\sigma^2}} & \text{if } z \geq 0. \end{cases}$$

What are the likelihood ratio tests for the binary hypothesis testing problem

$$\begin{array}{ll} H_1 : & Y \sim F_{\sigma_1^2} \\ H_0 : & Y \sim F_{\sigma_0^2} \end{array}$$

with $\sigma_0^2 \neq \sigma_1^2$, both strictly positive?
