ESTIMATION AND DETECTION THEORY

HOMEWORK # 2:

Please work out the **nine** (9) problems stated below – HVP refers to the text: H. Vincent Poor, *An Introduction to Signal Detection and Estimation* (Second Edition), Springer Texts in Electrical Engineering Springer, New York (NY), 2010. With this in mind, Exercise **II.2** (HVP) refers to Exercise 2 for Chapter II of HVP. Exercises are located at the end of each chapter.

Show work and **explain** reasoning.

1.

Solve Part (a) of Exercise **II.5** (HVP).

2.

Solve Part (a) of Exercise **II.6** (HVP).

3.

Solve Part (a) of Exercise **II.7** (HVP)

4.

Recall that a rv Z is said to be Rayleigh distributed with parameter $\sigma^2 > 0$ if its probability distribution F_{σ^2} admits a probability density function $f_{\sigma^2} : \mathbb{R} \to \mathbb{R}_+$ given by

$$f_{\sigma^2}(z) = \begin{cases} 0 & \text{if } z < 0 \\ \frac{z}{\sigma^2} e^{-\frac{z^2}{2\sigma^2}} & \text{if } z \geq 0. \end{cases}$$

What are the likelihood ratio tests for the binary hypothesis testing problem

$$\begin{array}{ll} H_1 : & Y_1, \ldots, Y_k \text{ i.i.d. with } Y_\ell \sim F_{\sigma_1^2}, \ \ell = 1, \ldots, k \\ H_0 : & Y_1, \ldots, Y_k \text{ i.i.d. with } Y_\ell \sim F_{\sigma_0^2}, \ \ell = 1, \ldots, k \end{array}$$

with $\sigma_0^2 \neq \sigma_1^2$, both strictly positive?

5.

In Problem **4** try to compute the probabilities $P_F(Lrt_\eta)$ and $P_M(Lrt_\eta)$ with $\eta > 0$. [**HINT:** What is the distribution of the rv Z^2 when Z is Rayleigh distributed with parameter $\sigma^2 > 0$?]

6.

You are facing the binary hypothesis testing problem

$$\begin{aligned} H_1 &: \quad Y \sim \mathrm{N}(1, \sigma^2) \\ H_0 &: \quad Y \sim \mathrm{N}(0, \sigma^2) \end{aligned}$$

with $\sigma^2 > 0$ and uniform prior on H. However, as you start thinking about its solution, you are told that the measurement Y is not available, and that you will have access only to $Z = Y^2$. Based on this modified measurement, obtain a decision rule which minimizes the probability of error criterion.

7.

In the context of Problem **6**, explore the loss of performance from using measurement Z instead of the original measurement Y.

8.

A rv Y is said to be a Bernoulli rv with parameter a (in $[0, 1]$) if

$$\mathbb{P}[Y = 1] = a \quad \text{and} \quad \mathbb{P}[Y = 0] = 1 - a.$$

Consider now the binary hypothesis testing problem

$$\begin{aligned} H_1 &: \quad Y_1, \ldots, Y_k \text{ i.i.d. with } Y_\ell \sim \mathrm{Ber}(a_1), \ \ell = 1, \ldots, k \\ H_0 &: \quad Y_1, \ldots, Y_k \text{ i.i.d. with } Y_\ell \sim \mathrm{Ber}(a_0), \ \ell = 1, \ldots, k \end{aligned}$$

with $a_1 \neq a_0$ in $(0, 1)$. What are the likelihood ratio tests for this binary hypothesis testing problem?

9.

In the context of Problem **8** can the Central Limit Theorem be used to compute $P_F(Lrt_\eta)$ and $P_M(Lrt_\eta)$ with $\eta > 0$ when k is large? Explain!