

**ESTIMATION AND DETECTION THEORY**

**HOMEWORK # 7:**

Please work out the **ten** (10) problems stated below – HVP refers to the text: H. Vincent Poor, *An Introduction to Signal Detection and Estimation* (Second Edition), Springer Texts in Electrical Engineering Springer, New York (NY), 2010. With this in mind, Exercise **II.2** (HVP) refers to Exercise 2 for Chapter II of HVP. Exercises are located at the end of each chapter.

**Show** work and **explain** reasoning.

**1.** \_\_\_\_\_

Solve Exercise **IV.5** (HVP).

**2.** \_\_\_\_\_

Solve Exercise **IV.6** (HVP).

**3.** \_\_\_\_\_

Solve Exercise **IV.8** (HVP).

**4.** \_\_\_\_\_

Solve Exercise **IV.11** Part (a) (HVP).

**5.** \_\_\_\_\_

Solve Exercise **IV.12** (HVP).

**6.** \_\_\_\_\_

Solve Exercise **IV.13** (HVP).

**7.** \_\_\_\_\_

Solve Exercise **IV.15** (HVP).

**8.** \_\_\_\_\_

Solve Exercise **IV.16** (HVP).

**9.** \_\_\_\_\_

The independent scalar observations  $Y_1, \dots, Y_k$  for some positive integer  $k$  all have finite mean  $m$  and variance  $\sigma^2$

**9.a** Consider the estimator  $g_1 : \mathbb{R}^k \rightarrow \mathbb{R}$  given by

$$g_1(y_1, \dots, y_k) = \frac{1}{k} \sum_{\ell=1}^k y_\ell, \quad y_\ell \in \mathbb{R}, \quad \ell = 1, \dots, k$$

Is this a biased estimator of  $m$  on the basis of  $Y_1, \dots, Y_k$ ?

**9.b** Consider now the estimator  $g_2 : \mathbb{R}^k \rightarrow \mathbb{R}$  given by

$$g_2(y_1, \dots, y_k) = \frac{1}{k} \sum_{\ell=1}^k (y_\ell - g_1(y_1, \dots, y_k))^2, \quad y_\ell \in \mathbb{R}, \quad \ell = 1, \dots, k$$

Is this an unbiased estimator  $\sigma^2$  on the basis of  $Y_1, \dots, Y_k$ ?

**10.** \_\_\_\_\_

Let  $Y_1, \dots, Y_n$  be i.i.d. Poisson rvs with parameter  $\theta > 0$ . Show that the statistic  $T : \mathbb{R}^n \rightarrow \mathbb{R}$  given by

$$T(y_1, \dots, y_n) = \sum_{i=1}^n y_i, \quad y_1, \dots, y_n \in \mathbb{R}$$

is a complete sufficient statistics for the family  $\{F_\theta^{(n)}, \theta > 0\}$  where  $F_\theta^{(n)}$  is the joint probability distribution of the rv  $(Y_1, \dots, Y_n)'$  under  $\mathbb{P}_\theta$ .

Find the MVUE for estimating  $\theta$  on the basis of  $(Y_1, \dots, Y_n)'$ . Carefully explain your reasoning!

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