ESTIMATION AND DETECTION THEORY

HOMEWORK # 6:

Please work out the **ten** (10) problems stated below – HVP refers to the text: H. Vincent Poor, *An Introduction to Signal Detection and Estimation* (Second Edition), Springer Texts in Electrical Engineering Springer, New York (NY), 2010. With this in mind, Exercise **II.2** (HVP) refers to Exercise 2 for Chapter II of HVP. Exercises are located at the end of each chapter.

Show work and **explain** reasoning.

1.

Solve Exercise **IV.1** (HVP).

2.

Solve Exercise **IV.2** (HVP).

3.

Solve Exercise **IV.3** (HVP).

4.

Solve Exercise **IV.4** (HVP).

5.

With θ in $(0,1)$, let F_θ be the probability distribution associated with the Binomial $\text{Bin}(m,\theta)$ for some positive integer $m \geq 1$. We are in the Bayesian framework, so that $Y \sim F_\theta$ is interpreted as

$$\mathbb{P}\left[Y = y | \vartheta = \theta\right] = \binom{m}{y} \theta^y (1-\theta)^{m-y}, \qquad \begin{array}{c} y = 0, \ldots, m \\ \theta \in (0,1). \end{array}$$

Show that the rv Y is uniformly distributed on $\{0,1,\ldots,m\}$ when $\vartheta \sim \mathcal{U}(0,1)$. [Hint: For $y = 0,\ldots,m-1$, relate $\mathbb{P}\left[Y = y+1\right]$ to $\mathbb{P}\left[Y = y\right]$ with the help of integration by parts!]

6.

With $\alpha < \beta$, write $\theta = (\alpha,\beta)$, and let F_θ denote the uniform distribution on the interval (α,β). Determine whether the family $\{F_\theta,\ \theta \in \Theta\}$ is an exponential family (with respect to Lebesgue measure) when

$$\Theta = \left\{\theta = (\alpha,\beta) \in [0,1]^2 :\ \alpha < \beta\right\}.$$