

ESTIMATION AND DETECTION THEORY

HOMEWORK # 6:

Please work out the **ten** (10) problems stated below – HVP refers to the text: H. Vincent Poor, *An Introduction to Signal Detection and Estimation* (Second Edition), Springer Texts in Electrical Engineering Springer, New York (NY), 2010. With this in mind, Exercise **II.2** (HVP) refers to Exercise 2 for Chapter II of HVP. Exercises are located at the end of each chapter.

Show work and **explain** reasoning.

1. _____
Solve Exercise **IV.1** (HVP).

2. _____
Solve Exercise **IV.2** (HVP).

3. _____
Solve Exercise **IV.3** (HVP).

4. _____
Solve Exercise **IV.4** (HVP).

5. _____
With θ in $(0, 1)$, let F_θ be the probability distribution associated with the Binomial $\text{Bin}(m, \theta)$ for some positive integer $m \geq 1$. We are in the Bayesian framework, so that $Y \sim F_\theta$ is interpreted as

$$\mathbb{P}[Y = y | \vartheta = \theta] = \binom{m}{y} \theta^y (1 - \theta)^{m-y}, \quad \begin{array}{l} y = 0, \dots, m \\ \theta \in (0, 1). \end{array}$$

Show that the rv Y is uniformly distributed on $\{0, 1, \dots, m\}$ when $\vartheta \sim \mathcal{U}(0, 1)$. [Hint: For $y = 0, \dots, m - 1$, relate $\mathbb{P}[Y = y + 1]$ to $\mathbb{P}[Y = y]$ with the help of integration by parts!]

6. _____
With $\alpha < \beta$, write $\theta = (\alpha, \beta)$, and let F_θ denote the uniform distribution on the interval (α, β) . Determine whether the family $\{F_\theta, \theta \in \Theta\}$ is an exponential family (with respect to Lebesgue measure) when

$$\Theta = \{\theta = (\alpha, \beta) \in [0, 1]^2 : \alpha < \beta\}.$$

7.

(Problem 6 continued) Is the family $\{F_\theta, \theta \in \Theta\}$ a complete family when

$$\Theta = \{\theta = (\alpha, \beta) \in [0, 1]^2 : \alpha < \beta\}?$$

Explain.

8.

An \mathbb{N} -valued rv is said to be a geometric rv with parameter ρ (in $(0, 1)$) if its pmf $\mathbf{p}_\rho \equiv \{p_\rho(y), y = 0, 1, \dots\}$ is given by

$$p_\rho(y) = (1 - \rho)\rho^y, \quad y = 0, 1, \dots$$

Is the family $\{\mathbf{p}_\rho, \rho \in (0, 1)\}$ a complete family? Explain.

9.

For some positive integer k , the observations Y_1, \dots, Y_k are \mathbb{R} -valued rvs given by

$$Y_i = \theta X + V_i, \quad i = 1, \dots, k$$

where θ is a scalar, and the rvs X, V_1, \dots, V_k are i.i.d. Gaussian rvs with mean zero and unit variance. Find the sufficient statistics of θ on the basis of Y_1, \dots, Y_k .

10.

(Problem 9 continued) For each θ in \mathbb{R} , let F_θ denote the probability distribution of the observation vector $(Y_1, \dots, Y_k)'$. Determine whether the family $\{F_\theta, \theta \in \mathbb{R}\}$ is an exponential family with respect to Lebesgue measure.
