ESTIMATION AND DETECTION THEORY

HOMEWORK # 7:

Please work out the **ten** (10) problems stated below – HVP refers to the text: H. Vincent Poor, *An Introduction to Signal Detection and Estimation* (Second Edition), Springer Texts in Electrical Engineering Springer, New York (NY), 2010. With this in mind, Exercise **II.2** (HVP) refers to Exercise 2 for Chapter II of HVP. Exercises are located at the end of each chapter.

Show work and **explain** reasoning.

1.

Solve Exercise **IV.5** (HVP).

2.

Solve Exercise **IV.6** (HVP).

3.

Solve Exercise **IV.8** (HVP).

4.

Solve Exercise **IV.11** Part (a) (HVP).

5.

Solve Exercise **IV.12** (HVP).

6.

Solve Exercise **IV.13** (HVP).

7.

Solve Exercise **IV.15** (HVP).

8.

Solve Exercise **IV.16** (HVP).

9.

The independent scalar observations $Y_1, \ldots, Y_k$ for some positive integer k all have finite mean m and variance σ^2

9.a Consider the estimator $g_1 : \mathbb{R}^k \to \mathbb{R}$ given by

$$g_1(y_1, \ldots, y_k) = \frac{1}{k} \sum_{\ell=1}^{k} y_\ell, \qquad \begin{array}{c} y_\ell \in \mathbb{R}, \\ \ell = 1, \ldots, k \end{array}$$

Is this a biased estimator of m on the basis of $Y_1, \ldots, Y_k$?

9.b Consider now the estimator $g_2 : \mathbb{R}^k \to \mathbb{R}$ given by

$$g_2(y_1, \ldots, y_k) = \frac{1}{k} \sum_{\ell=1}^{k} \left(y_\ell - g_1(y_1, \ldots, y_k) \right)^2, \qquad \begin{array}{c} y_\ell \in \mathbb{R}, \\ \ell = 1, \ldots, k \end{array}$$

Is this an unbiased estimator σ^2 on the basis of $Y_1, \ldots, Y_k$?

10.

Let $Y_1, \ldots, Y_n$ be i.i.d. Poisson rvs with parameter $\theta > 0$. Show that the statistic $T : \mathbb{R}^n \to \mathbb{R}$ given by

$$T(y_1, \ldots, y_n) = \sum_{i=1}^{n} y_i, \quad y_1, \ldots, y_n \in \mathbb{R}$$

is a complete sufficient statistics for the family $\{F_\theta^{(n)}, \ \theta > 0\}$ where $F_\theta^{(n)}$ is the joint probability distribution of the rv $(Y_1, \ldots, Y_n)'$ under $\mathbb{P}_\theta$.

Find the MVUE for estimating θ on the basis of $(Y_1, \ldots, Y_n)'$. Carefully explain your reasoning!
