ENEE 630 Homework 2[1]

Material Covered: Polyphase Representation, Uniform DFT Filter Banks, Analysis/Synthesis system

Problem 1 Consider the structure shown in Fig P-1(a), where W is the 3×3 DFT matrix.

This is a three channel synthesis bank with three filters $F_0(z)$, $F_1(z)$, $F_2(z)$. (For example $F_0(z) = Y(z)/Y_0(z)$ with $y_1(n)$ and $y_2(n)$ set to zero.)

a) Let $R_0(z) = 1 + z^{-1}$, $R_1(z) = 1 - z^{-2}$, $R_2(z) = 2 + 3z^{-1}$. Find expressions for the three synthesis filters $F_0(z)$, $F_1(z)$, $F_2(z)$.

b) The magnitude response of $F_1(z)$ is sketched in Fig. P-1(b). Plot the responses $|F_0(e^{j\omega})|$ and $|F_2(e^{j\omega})|$. Does the relation between $F_0(z)$, $F_1(z)$, and $F_2(z)$ depend on choices of $R_k(z)$?

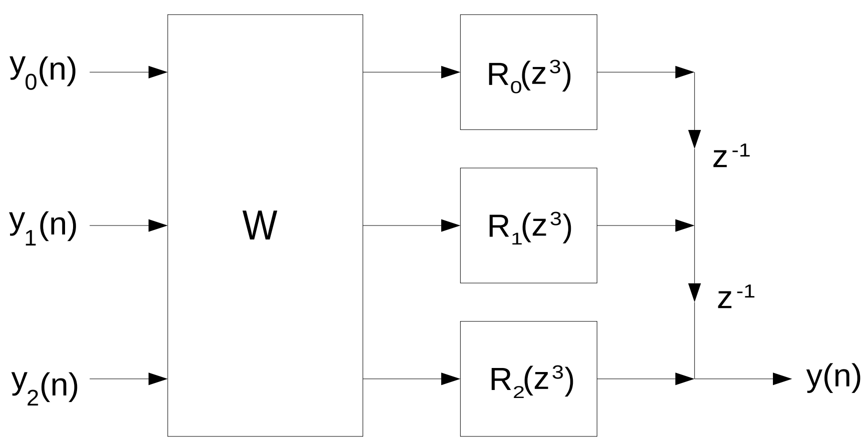

Figure: P-1(a)

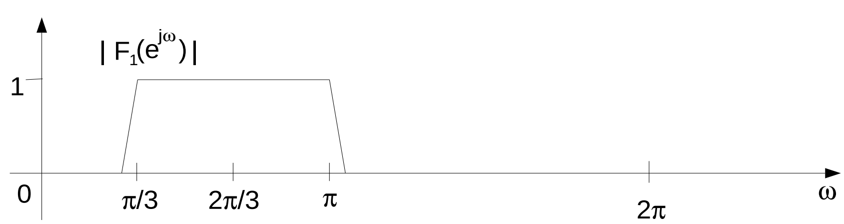

Figure: P-1(b)

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Problem 2 For a uniform DFT analysis bank, we know that the filters are related by $H_k(z) = H_0(zW^k)$, $0 \leq k \leq M-1$, with $W = e^{-j2\pi/M}$. Let $M = 5$ and define two new transfer functions $G_1(z) = H_1(z) + H_4(z)$ and $G_2(z) = H_2(z) + H_3(z)$. Let $h_0(n)$ denote the impulse response of $H_0(z)$, assumed to be real for all n.

a) Are $h_k(n)$, $1 \leq k \leq 4$ real for all n?

b) Express the impulse responses $g_1(n)$ and $g_2(n)$ of $G_1(z)$ and $G_2(z)$ in terms of $h_0(n)$. Are $g_1(n)$ and $g_2(n)$ real for all n?

c) Let $|H_0(e^{j\omega})|$ be as shown in Fig. P-2. Plot the responses $|G_1(e^{j\omega})|$ and $|G_2(e^{j\omega})|$, for $0 \leq \omega \leq 2\pi$. Does $|G_2(e^{j\omega})|$ necessarily look 'good' in its passband?

Figure : P-2

Problem 3 Let $H(z) = \sum_{n=0}^{N} h(n) z^{-n}$ with $h(n) = h(N-n)$. Consider the Type-1 polyphase representation for $H(z)$ with M polyphase components. The symmetry of $h[n]$ reflects into the coefficients of the polyphase components $E_l(z)$ as follows: there exists an integer m_0 (with $0 \leq m_0 \leq M-1$) such that $e_k[n]$ is the image of $e_{m_0-k}[n]$ for $0 \leq k \leq m_0$, and $e_k[n]$ is the image of $e_{M+m_0-k}[n]$ for $m_0 + 1 \leq k \leq M-1$.

a) Take an example of a 7^{th}-order H(z) and verify the above statement for $M = 3$. What is m_0? How about when $M = 4$?

b) Prove the above statement. Find out how m_0 is related to N and M.

Problem 4 Consider the analysis/synthesis system in Fig. P-4.

a) Let the analysis filters be $H_0(z) = 1 + 3z^{-1} + 0.5z^{-2} + z^{-3}$ and $H_1(z) = H_0(-z)$. Find causal stable IIR fitlers $F_0(z)$ and $F_1(z)$ such that $\hat{x}(n)$ agrees with $x(n)$ except for a possible delay and (nonzero) scale factor.

b) Let $H_0(z) = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$, and $H_1(z) = H_0(-z)$. Find causal FIR fitlers $F_0(z)$ and $F_1(z)$ such that $\hat{x}(n)$ agrees with $x(n)$ except for a possible delay and (nonzero) scale factor.

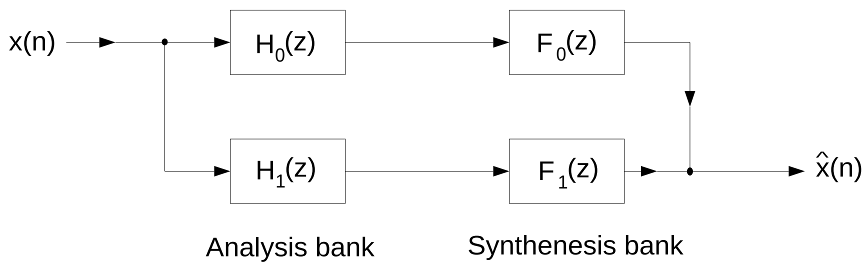

Figure : P-4

(Hint: Polyphase decomposition may help to solve this problem. Review of complementary filters might help as well. And there is a counter part of Euclid's theorem in polynomial form for two relatively prime polynomials.)