ENEE 630 Homework 3[1]

Material Covered: Uniform and Nonuniform QMF Filters, Perfect Reconstruction

Problem 1 In a two-channel QMF bank, if the filters are related as

$$H_1(z) = H_0(-z), F_0(z) = H_0(z), F_1(z) = -H_1(z)$$

and if $H_0(z)$ is chosen to have real coefficients and linear phase, then the distortion function is given by

$$T(e^{jw}) = \frac{1}{2}e^{-jwN}(|H_0(e^{jw})|^2 - (-1)^N|H_0(e^{j(\pi-w)})|^2) \quad (1)$$

If filter order N is even, this implies $T(e^{j\frac{\pi}{2}}) = 0$, so in order to avoid strong attenuation on $\omega = \frac{\pi}{2}$, the filter order N for this filter bank structure has to be odd for many applications. Now consider a modified QMF bank shown in Figure P-1 where the filters could be FIR or IIR. Express $\hat{X}(z)$ in terms of X(z). With $H_1(z) = H_0(-z)$, show that the choice $F_0(z) = H_0(z)$ and $F_1(z) = H_1(z)$ cancels aliasing. With this choice write down the distortion T(z) in terms of $H_0(z)$.

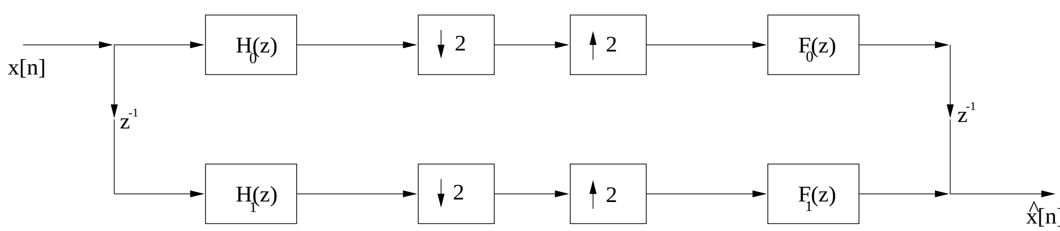

Figure : P-1

a) Now let $H_0(z)$ be a real coefficient linear phase FIR lowpass filter of order N. Simplify T(z) and show that there is no phase distortion. Also show that N has to be even in order to avoid the condition $T(e^{j\frac{\pi}{2}}) = 0$.

b) For the system in part (a) with N even, what is the number of MPU's required to implement the analysis bank? How does this compare with the numbers we obtained for the case of traditional two-channel QMF with odd N?
Hint: Try to exploit as many of the following facts as you can – (i) the relation $H_1(z) = H_0(-z)$, (ii) the linear phase property, and (iii) the presence of decimators.

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Problem 2 Suppose the analysis filters in a two-channel QMF bank are given by the following set of equations:

$$H_0(z) = 2 + 6z^{-1} + z^{-2} + 5z^{-3} + z^{-5}$$
$$H_1(z) = H_0(-z)$$

Find a set of stable synthesis filters which result in perfect reconstruction.

Problem 3 Given below are three sets of FIR analysis banks for a 3-channel maximally decimated QMF bank. In each case, answer the following: (i) Is it possible to obtain a set of FIR synthesis filters for perfect reconstruction? If so, find them. (ii) If not, find a set of IIR synthesis filters for perfect reconstruction. (iii) In the latter event, are the synthesis filters stable?

a) $H_0(z) = 1, H_1(z) = 2 + z^{-1}, H_2(z) = 3 + 2z^{-1} + z^{-2}$

b) $H_0(z) = 1, H_1(z) = 2 + z^{-1} + z^{-5}, H_2(z) = 3 + 2z^{-1} + z^{-2}$

c) $H_0(z) = 1, H_1(z) = 2 + z^{-1} + z^{-5}, H_2(z) = 3 + z^{-1} + 2z^{-2}$

Problem 4 Tree structures can be used to obtain QMF banks in which the decimation ratio is not the same of all channels (called nonuniform filter banks). Consider the system shown in Fig. P-4(a). This is equivalent to the system in Fig. P-4(b).

a) Identify the filters $G_k(z)$ and $P_k(z)$ in terms of the filters $H_0(z), H_1(z), F_0(z)$ and $F_1(z)$. Suppose that $H_0(z)$ and $H_1(z)$ are real coefficient filters with magnitude responses as shown in Fig. P-4(c). Sketch the magnitude responses $|G_k(e^{jw})|$ for $0 \leq k \leq 4$. Thus, the filters have unequal bandwidths, and the decimation ratios are inversely proportional to these bandwidths. This is called nonuniform (maximally decimated) QMF bank.

b) Suppose the filters $H_0(z), H_1(z), F_0(z)$ and $F_1(z)$ are such that the traditional two channel QMF bank has perfect reconstruction property, with distortion function T(z)=1. Show that the five-channel nonuniform system also has perfect reconstruction property.

c) Suppose the filters $H_0(z), H_1(z), F_0(z)$ and $F_1(z)$ are such that the traditional two-channel QMF bank is alias-free, with distortion function T(z). Does the above five-channel nonuniform system remain alias-free? If not, how would you modify the structure of Fig. P-4(a) to obtain this property, and what is the resulting distortion function?

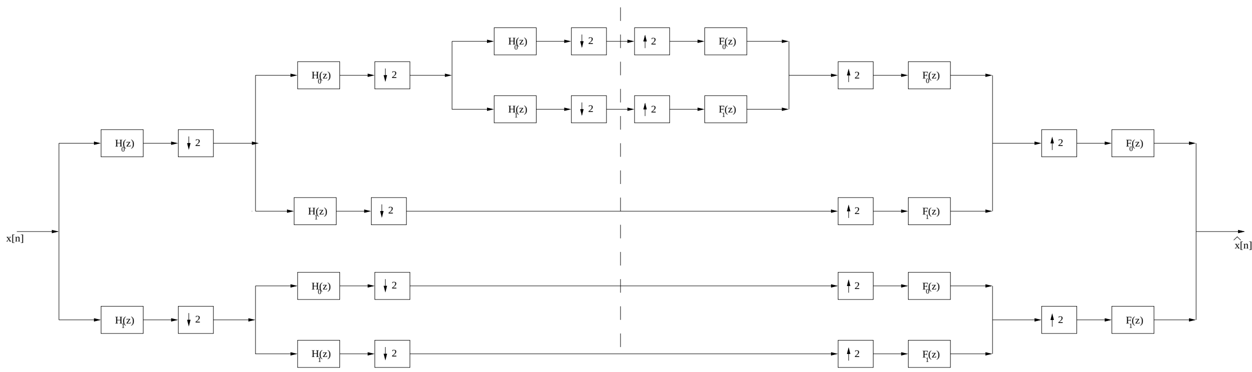

Figure : P-4(a)

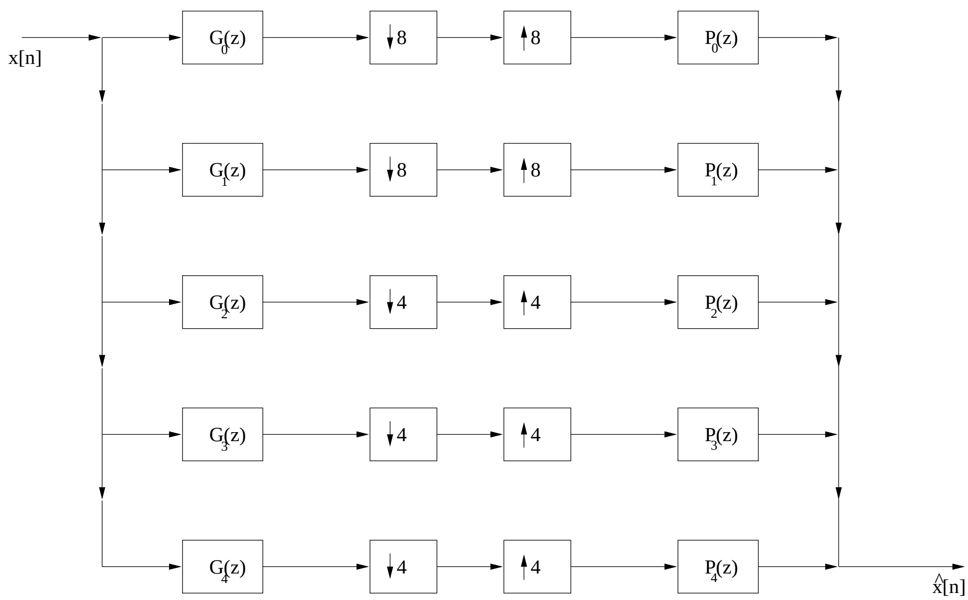

Figure : P-4(b)

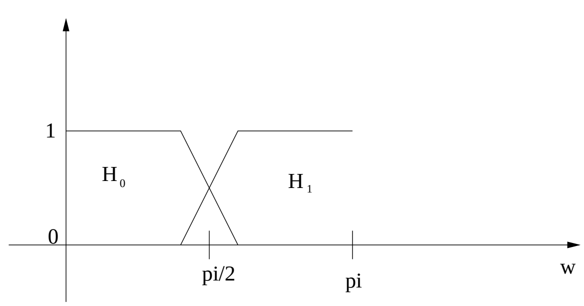

Figure : P-4(c)