ENEE 630 Homework 5[1]

Material Covered: Auto-Regressive (AR), Moving-Average (MA) and ARMA Process. Yule-Walker Equations, Autocorrelation Functions.

Problem 1 A first order autoregressive (AR) process {u(n)} that is real-valued satisfies the real-valued difference equation

$$u(n) + a_1 u(n-1) = v(n)$$

where a_1 is a constant and {v(n)} is a white-noise process of varance σ_v^2. Such a process is also referred to as a *first-order Markov process*.

(a) Suppose in practical implementation, the generation of the process {u(n)} starts at n=1 with initialization u(0)=0. Determine the mean of the actual {u(n)} process that we have obtained. Under what conditions $E[u(n)]$ converges to a constant and what the constant is?

(b) Now consider the case when {v(n)} has zero mean. Determine the variance of the actual {u(n)} process that we have obtained. Under what conditions $Var[u(n)]$ converges to a constant and what the constant is?

(c) For the conditions specified in part (b), find the autocorrelation function of the AR process {u(n)}. Sketch this autocorrelation function when $n \gg k$, for the two cases $0 < a_1 < 1$ and $-1 < a_1 < 0$.

Problem 2 Consider an autoregressive process {u(n)} of order 2, described by the difference equation

$$u(n) = u(n-1) - 0.5u(n-2) + v(n)$$

where {v(n)} is a white-noise process of zero mean and variance 0.5

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(a) Write the Yule-Walker equations for the process.

(b) Solve these two equations for the autocorrelation function values r(1) and r(2).

(c) Find the variance of {u(n)}.

Problem 3 Consider an MA process {x(n)} of order 2 described by the difference equation

$$x(n) = v(n) + 0.75v(n-1) + 0.25v(n-2)$$

where {v(n)} is a zero mean white noise process of unit variance. The requirement is to approximate the process by an AR process {u(n)} of order M. Do this approximation for the orders $M = 2$ and $M = 5$, respectively.

Problem 4 A discrete-time stochastic process {x(n)} that is real-valued consists of an AR process {u(n)} and additive white noise process $\{v_2(n)\}$. The AR component is described by the difference equation

$$u(n) + \sum_{k=1}^{M} a_k u(n-k) = v_1(n)$$

where $\{a_k\}$ are the set of AR parameters and $\{v_1(n)\}$ is a white noise process that is independent of $\{v_2(n)\}$. Show that {x(n)} is an ARMA process described by

$$x(n) = -\sum_{k=1}^{M} a_k x(n-k) + \sum_{k=1}^{M} b_k e(n-k) + e(n)$$

where {e(n)} is a white noise process. How are the MA parameters $\{b_k\}$ defined? How is the variance of e(n) defined?