ENEE630 ADSP **RECITATION 2 w/ solution** **September 13/14, 2012**

1. Consider the structures shown in Fig. RI.1, with input transforms and filter responses as indicated. Sketch the quantities $Y_0(e^{j\omega})$ and $Y_1(e^{j\omega})$.

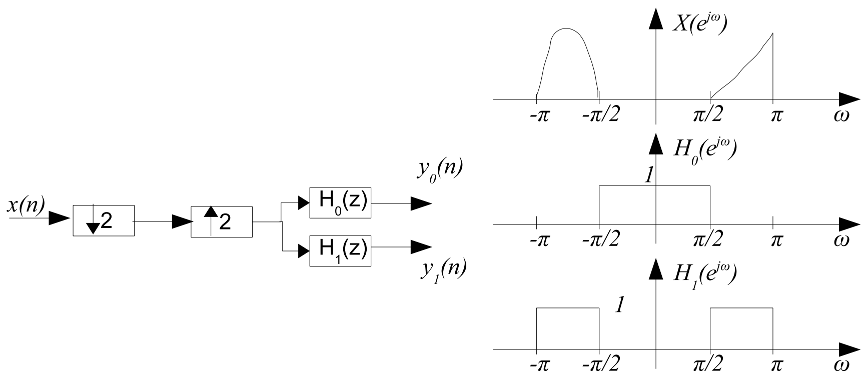
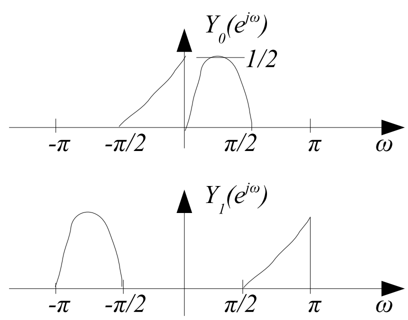


Figure RI.1:

Solution:



Comment: For a down-sampled signal, it's still possible to recover the original signal (not necessarily low-pass) using filters and multirate building blocks as long as there is no aliasing (H_1 in this problem).

2. For each case shown in the Fig. RI.2, prove or disprove whether the left system is equivalent to the right system? Assume M, L, K are all integers larger than 1.

Solution: Assume the input, output, intermediate signals are $x(n), y(n)$, and $u(n)$, respectively.

(a). (FD) $U(z) = X(z^M)$, $Y_2(z) = \frac{1}{M}\sum_{k=0}^{M-1} U(z^{1/M}W_M^k) = \frac{1}{M}\sum_{k=0}^{M-1} X(zW_M^{kM}) = X(z)$.

(TD) $u(n) = x(\frac{n}{M})$ if n is a multiple of M, and $u(n) = 0$ otherwise. Then, $y_2(n) = u(Mn) = x(n)$.

(b). (FD) $U(z) = \frac{1}{M}\sum_{k=0}^{M-1} X(z^{1/M}W_M^k)$, $Y_2(z) = U(z^M) = \frac{1}{M}\sum_{k=0}^{M-1} X(zW_M^k)$.

(TD) $u(n) = x(Mn)$, $y_2(n) = u(\frac{n}{M}) = x(n)$ only if n is a multiple of M, i.e., $y_2(n) = \begin{cases} x(n) & n \text{ is multiple of } M \\ 0 & \text{otherwise.} \end{cases}$

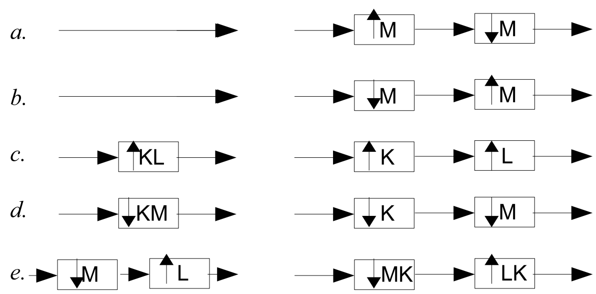


Figure RI.2:

(c). (FD) $U(z) = X(z^K)$, $Y_2(z) = U(z^L) = X(z^{KL})$.
(TD) $u(n) = x(\frac{n}{K})$ only if n is a multiple of K, and $y_2(n) = u(\frac{n}{L})$ only if n is a multiple of L. Zero otherwise. Hence, $y_2(n) = x(\frac{n}{KL})$ only when n is a multiple of KL.

(d) (FD) $U(z) = \frac{1}{K}\sum_{k=0}^{K-1} X(z^{1/K}W_K^k)$, $Y_2(z) = \frac{1}{M}\sum_{m=0}^{M-1} U(z^{1/M}W_M^m)$
$= \frac{1}{MK}\sum_{m=0}^{M-1}\sum_{k=0}^{K-1} X(z^{1/KM}e^{-j\frac{2\pi m}{MK}}e^{-j\frac{2\pi kM}{MK}}) = \frac{1}{MK}\sum_{m=0}^{M-1}\sum_{k=0}^{K-1} X(z^{1/KM}W_{MK}^{kM+m})$
$= \frac{1}{MK}\sum_{t=0}^{MK-1} X(z^{1/KM}W_{MK}^t)$
(TD) $u(n) = x(Kn)$, $y_2(n) = u(Mn) = x(KMn)$.

(e) (FD) Left: $Y_1(z) = \frac{1}{M}\sum_{k=0}^{M-1} X(z^{L/M}W_M^k)$; Right: $Y_2(z) = \frac{1}{MK}\sum_{k=0}^{MK-1} X(z^{L/M}W_{MK}^k)$
(TD) Left: $y_2(n) = x(n\frac{M}{L})$ only when n is a multiple of L; right: $y_2(n) = x(n\frac{M}{L})$ only when n is a multiple of KL.

Note: To prove/disprove the equivalence, you can prove either in time domain or in freq domain, no need to do both. You are recommended to try all on your own (except (d) freq domain) to familiar yourself with the derivation.

3. Simplify the following systems in Fig. I.3.

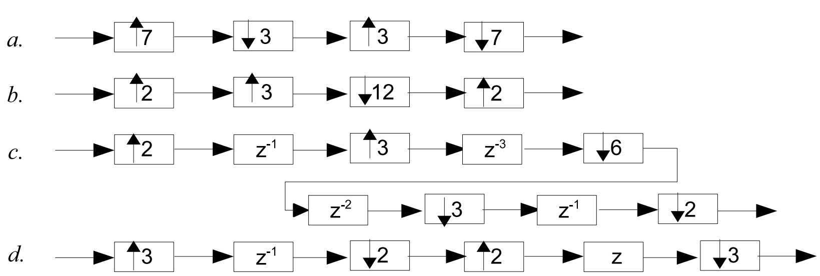
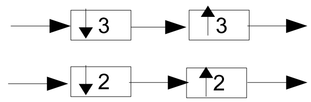


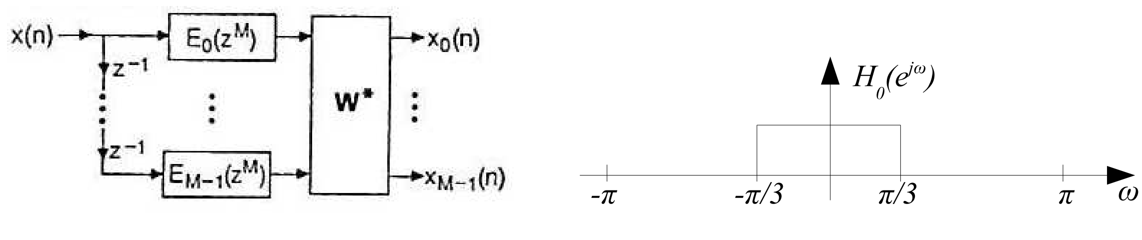
Figure RI.3:

Solution: The simplified systems are as follows

a. 

b.

5. A uniform DFT analysis bank (Type 1) is shown in Fig. RI.5(a), where $\boldsymbol{W}^*$ is the $M \times M$ IDFT matrix, i.e., the (m,n)-th entry is W_M^{-mn} with indices m, n starting from 0. The transfer function from input port $x(n)$ to output port $x_k(n)$ is denoted by $H_k(z)$. Answer the following questions.



(a) (b)

Figure RI.5:

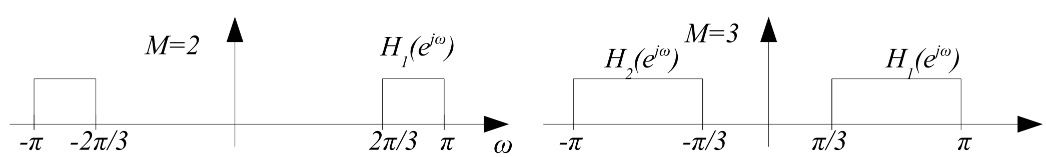
(a). Prove $H_k(z) = H_0(zW_M^k)$ for $0 \le k \le M-1$. Given $H_0(e^{j\omega})$ in Fig. RI.5(b), sketch $H_1(e^{j\omega})$ when $M = 2$, and $H_1(e^{j\omega})$, $H_2(e^{j\omega})$ when $M = 3$.

(b). $M = 4$. Assume $E_0(z) = 1+z^{-1}$, $E_1(z) = 1+2z^{-1}$, $E_2(z) = 2+z^{-1}$, and $E_3(z) = 0.5+z^{-1}$. Find numerical values of these filter coefficients for $H_k(z)$, $0 \le k \le 3$.

(c). $M = 2$. Let $H_0(z) = 1 + 2z^{-1} + 4z^{-2} + 2z^{-3} + z^{-4}$, and let $H_1(z) = H_0(-z)$. Draw an implementation for the pair $[H_0(z), H_1(z)]$ in the form of a uniform DFT analysis bank, explicitly showing the polyphase components, the 2×2 IDFT box, and other relevant details.

Solution:

(a). $H_0(z) = \sum_{m=0}^{M-1} z^{-m} E_m(z^M)$, $H_k(z) = \sum_{m=0}^{M-1} z^{-m} E_m(z^M) W_M^{-mk} = \sum_{m=0}^{M-1} (zW_M^k)^{-m} E_m((zW_M^k)^M) = H_0(zW_M^k)$. $H_k(e^{j\omega}) = H_0(e^{j(\omega - 2\pi k/M)})$.



(b). $H_0(z) = 1 + z^{-1} + 2z^{-2} + 0.5z^{-3} + z^{-4} + 2z^{-5} + z^{-6} + z^{-7}$. $H_1(z) = 1 + jz^{-1} - 2z^{-2} - 0.5jz^{-3} + z^{-4} + 2jz^{-5} - z^{-6} - jz^{-7}$. $H_2(z) = 1 - z^{-1} + 2z^{-2} - 0.5z^{-3} + z^{-4} - 2z^{-5} + z^{-6} - z^{-7}$. $H_3(z) = 1 - jz^{-1} - 2z^{-2} + 0.5jz^{-3} + z^{-4} - 2jz^{-5} - z^{-6} + jz^{-7}$. ($j = \sqrt{-1}$).

Note:

1. $W_M^1 = e^{-j\frac{2\pi}{M}}$, e.g., $W_2^1 = -1$, $W_4^1 = -j$.

2. $\boldsymbol{W}^*$ (with entry W_M^{-ij}) is called the IDFT matrix, whereas $\boldsymbol{W}$ (with entry W_M^{ij}) is called the DFT matrix. Note that the index starts from 0 and goes up to $M-1$. One important property is

that $\boldsymbol{W}^*\boldsymbol{W} = M\boldsymbol{I}$. For instance, when $M = 2$, $\boldsymbol{W}^*$ and $\boldsymbol{W}$ happen to be the same

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix},$$

and hence using the property,

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},$$

which you will encounter very often when studying the QMF bank.