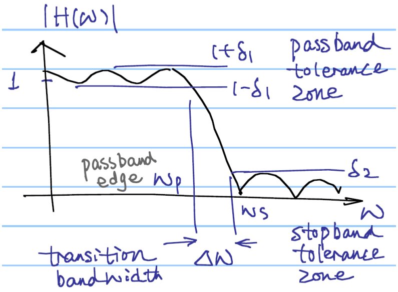
Multi-rate Signal Processing
4. Multistage Implementations
5. Multirate Application: Subband Coding

Electrical & Computer Engineering
University of Maryland, College Park

Acknowledgment: ENEE630 slides were based on class notes developed by Profs. K.J. Ray Liu and Min Wu. The LaTeX slides were made by Prof. Min Wu and Mr. Wei-Hong Chuang.

Contact: *minwu@umd.edu*. Updated: September 16, 2012.

Preliminaries: Filter's magnitude response



Filter design theory

A linear phase FIR filter that satisfies this specification has order $N = g(\delta_1, \delta_2, \Delta W)$

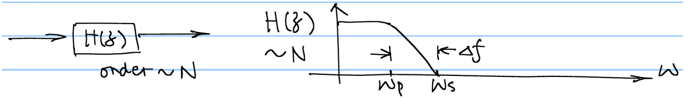
- as a function of δ_1, δ_2, and Δf
- $\Delta_f \approx \frac{\Delta W}{2\pi}$ (normalized transition b.w. $\in [0, 1]$)

For fixed ripple size, $N \propto \frac{1}{\Delta f}$:

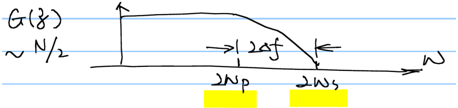
$\Delta f \uparrow \rightarrow N \downarrow$ (computation $\downarrow$)

Doubling Filter Transition Band

Consider an original LPF implementation



If we have a LPF with transition band $2\Delta f$, we may reduce the order by about half.



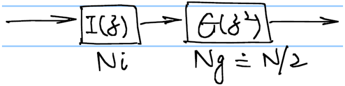
Double transition band leads to half of the required order for the filter.

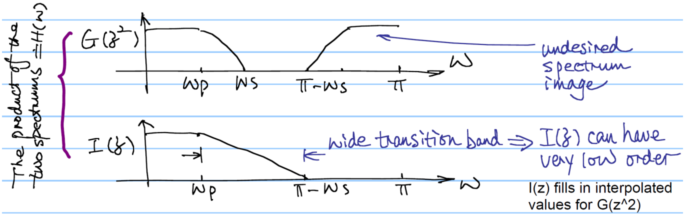
Interpolated FIR (IFIR)

Questions:

- With passband and stopband also doubled, what will be the response of a new filter that is an expanded version of the impulse response for $G(z)$, i.e., $G(z^2)$?
- What else is needed to get the same system response as $H(z)$?

New Interpolated FIR Design:





Multistage Decimation / Expansion

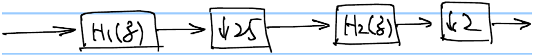
With what we have in IFIR design, reconsider now the efficient implementation of multirate filters:

e.g.,

$M = 50$

- Narrow passband for $H(z)$
  $\Rightarrow$ long filter needed
- Using polyphase representation
  $\Rightarrow$ need many decomposition components for large M!

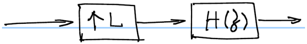
How about?



Multistage implementation can be more efficient (in terms of computations per unit time).

Multistage Decimation / Expansion

Similarly, for interpolation,



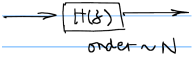
Summary

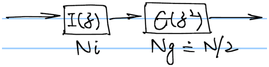
By implementing in multistage, not only the number of polyphase components reduces, but most importantly, the filter specification is less stringent and the overall order of the filters are reduced.

Exercises:

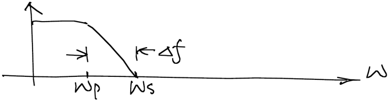
- Close book and think first how you would solve the problems.
- Sketch your solutions on your notebook.
- Then read V-book Sec. 4.4.

IFIR Design

Original system: 

New system: 

$H(z) \sim N$

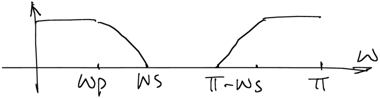


(omit ripples in the sketches)

$G(z) \sim \frac{N}{2}$

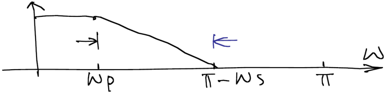
Doubled transition band leads to half of the required order for the filter

$G(z^2)$



Note the undesired spectrum image

$I(z)$



Wide transition band $\Rightarrow$ $I(z)$ can have very low order

※ $G(2\omega) \times I(\omega) \approx H(\omega)$

Discussions

The complexity of the two-stage implementation is much less than that of the direct implementation.

- $G(z)$: the model filter
  (designed according to the “scaled” specification of $H(z)$)
- $I(z)$: image suppressor
- Number of adders: $N_i + N_g \ll N$
- Number of multipliers: $(N_i + 1) + (N_g + 1) \ll (N + 1)$

Principle of IFIR Design

$\Rightarrow$ Motivated multistage design from an efficient design technique of narrowband LPF known as IFIR.

- Applicable for designing any narrowband FIR filter (by itself not tied with $\uparrow L$ or $\downarrow M$)

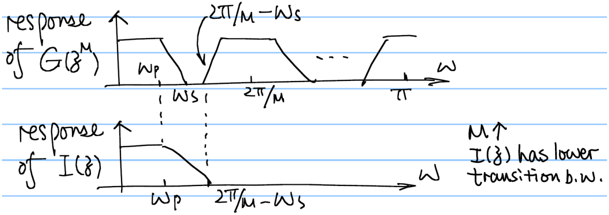
Readings: Vaidyanathan's Book Sec. 4.4

Extension to $M \geq 2$

In general, it is possible to stretch more, by an amount $M \geq 2$,

- so that the transition band of $G(z)$ can be even wider ($\approx M\Delta f$) and further reduces the order N_g
- Stopband edge in $G(z)$: $M\omega_s \leq \pi$
  $\Rightarrow M \leq \lfloor \frac{\pi}{\omega_s} \rfloor$

Extension to $M \geq 2$: Tradeoff

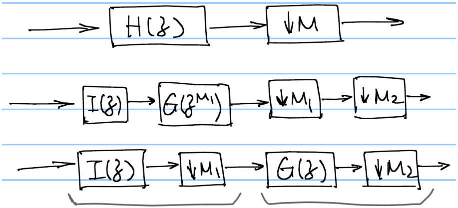


Tradeoff of the total cost: $M \uparrow$

- $G(z)$: transition b.w. $\uparrow \rightarrow$ order $\downarrow$
- $I(z)$: transition b.w. $\downarrow$ (could become very narrow) $\rightarrow$ order $\uparrow$

$\Rightarrow$ can search for optimal M.

Multistage Design of Decimation Filter



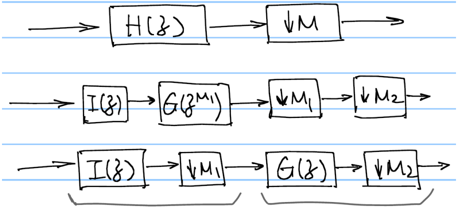
polyphase implementation each stage

$M = M_1 M_2$:

Choice of M_1 can be cast as an optimization problem

Rule of thumb: choose M_1 larger to reduce the computation complexity & data rate early on

Multistage Design Example: (1) Direct Design



polyphase implementation each stage

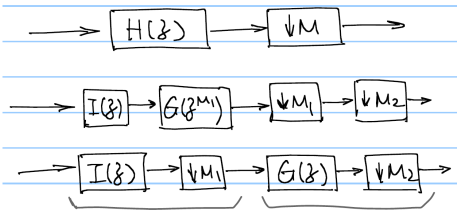
e.g., $M = 50$ fold decimation of an $8k$Hz signal

$H(z)$: $\delta_1 = 0.01$, $\delta_2 = 0.001$, passband edge = 70Hz, stopband edge = 80Hz

$\sim$ normalized $\Delta f = \frac{10}{8k} = \frac{1}{800}$

the order of direct equiripple filter design $\Rightarrow N = \mathbf{2028}$

Multistage Design Example: (2) Two-stage Design



polyphase implementation each stage

$M_1 = 25,\ M_2 = 2$

$G(z):\ \Delta f = 25 \times \frac{1}{800}$
$\omega_p = 0.4375\pi, \omega_s = 0.5\pi,$
$\delta_1 = 0.005, \delta_2 = 0.001$

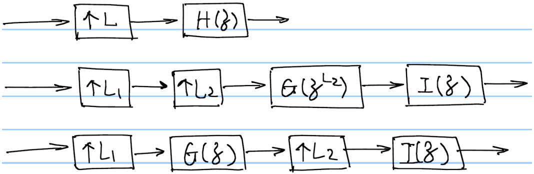
$\Rightarrow N_g = \mathbf{90}$

$I(z):\ \Delta f = 17 \times \frac{1}{800}$
$\omega_p = 0.0175\pi, \omega_s = 0.06\pi,$
$\delta_1 = 0.005, \delta_2 = 0.001$

$\Rightarrow N_i = \mathbf{139}$
higher order than $G(z)$ due to narrower transition

See spectrum sketch in Vaidyanathan's Book, Fig. 4.4-6.

Interpolation Filter



L_1 should be small to avoid too much increase in data rate and filter computation at early stage

e.g., $L = 50$: $L_1 = 2$, $L_2 = 25$

Summary

By implementing in multistage, not only the number of polyphase components reduces, but most importantly, the filter specification is less stringent and the overall order of the filters are reduced.

5.1 Multirate Applications in Digital Audio Systems

- During A/D conversion: Oversampling to alleviate the stringent requirements on the analog anti-aliasing filter
- During D/A conversion: Filter to remove spectrum images
- Fractional sampling rate conversion: Studio 48KHz vs. CD 44.1KHz

Readings to explore more: Vaidynathan Tutorial Sec. III-A.

5.2 Subband Coding: How to compress a signal?

- Tradeoff between bit rate and fidelity
- Many aspects to explore:
  use bits wisely; exploit redundancy; discard unimportant parts; ...
- Allocate bit rate strategically: equal allocation vs. focused effort

Compression Tool #1 (lossless if free from aliasing): Downsample a signal of limited bandwidth

(From what we learned about decimation in §1.1)

If a discrete-time signal is bandlimited with bandwidth smaller than 2π, the signal can be decimated by an appropriate factor without losing information.

- i.e., we don't need to keep that many samples
- Recall the example in §1.1.1: $|\omega| < \frac{2}{3}\pi$
  $\Rightarrow$ can change data rate to $\frac{2}{3}$ of original
- If signal spectrum support is in $(\omega_1, \omega_1 + \frac{2\pi}{M})$, we can decimate the signal by M fold without introducing aliasing.
  (Decimated signal may extend to entire 2π spectrum range)

Compression Tool #2 (lossy): Quantization

Dynamic range A of a signal: the value range

- Use a finite number of bits to represent a continuous valued sample via scalar quantization:

  partition A into N intervals, pick N representative values and use $\log_2 N$ bits to represent each value.

  $\rightarrow$ Simple quantization: uniform quantization

Compression Tool #2 (lossy): Quantization

- Quantify the "imprecision" between original and quantized:
  - maximum error $\max_x |x - \hat{x}|$
  - mean squared error $\mathbb{E}[(x - \hat{x})^2]$: easy to differential in an optimization formulation

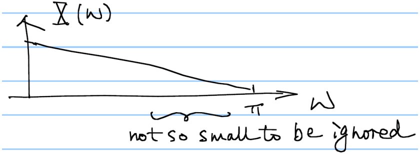
- For a fixed amount of average error, signal with large dynamic range requires more bits in representation.
  e.g., uniform quantizer: max error $= A/(2N)$
  $\Rightarrow$ dynamic range $A \uparrow$ or # intervals $N \downarrow$ lead to higher error

- Non-uniform quantizer: may consider a few aspects
  1. keep relative error low (smaller stepsize in low value range)
  2. take account of signal's probability distribution and keep the expected error low (reduce error in most seen values)
     e.g., MMSE / Lloyd-Max quantizer

Non-bandlimited Signals

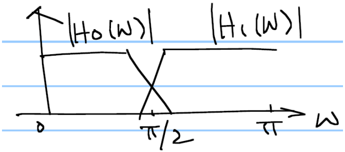
We often encounter signals that are not bandlimited, but have dominant frequency bands.

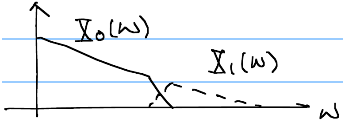
Question: How to use fewer bits to represent the signal and keep the imprecision low?



e.g. $x[n]$: 10kHz sampled signal, 16 bits/sample (to cover the dynamic range) $\Rightarrow$ data bit rate 160$kbps$

Subband Coding





1. $x_0[n]$ and $x_1[n]$ are bandlimited and can be decimated
2. $\mathbb{X}_1(\omega)$ has smaller power s.t. $x_1[n]$ has smaller dynamic range, thus can be represented with fewer bits

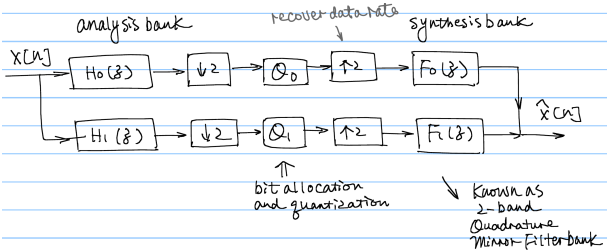
Suppose now to represent each subband signal, we need

$x_0[n]$: 16 bits / sample

$x_1[n]$: 8 bits / sample

$\therefore 16 \times \frac{10k}{2} + 8 \times \frac{10k}{2} = 120k\text{bps}$

Filter Bank for Subband Coding



Role of $F_k(z)$:

- Eliminate spectrum images introduced by $\uparrow 2$, and recover signal spectrum over respective freq. range
- If $\{H_k(z)\}$ is not perfect, the decimated subband signals may have aliasing.
- $\{F_k(z)\}$ should be chosen carefully so that the aliasing gets canceled at the synthesis stage (in $\hat{x}[n]$).

Warm-up Exercise: Two-Channel Filter Bank

Under what conditions does a filter bank preserve information?

Derive the input-output relation in Z-domain.