Electrical and Computer Engineering Department
University of Maryland
College Park, Maryland

ENEE 660 – System Theory

Final Exam, 10:30 a.m.-12:30 p.m.
Saturday, December 18, 2010
Room 3118 CSI

Closed book exam. Answer any six questions.

Problem 1

Find a control $u(\cdot)$ that transfers the system

$$\dot{x}(t) = b(t)\ u(t) \qquad\qquad b(t) \neq 0$$

from the state $x(0) = x_0$ to the state $x(1) = x_1$ *and minimizing*

$$J[u] = \int_0^1 q(t)\ u^2(t)\ dt$$

where $q(t) > 0$ is given and when,

(a) $x_0 = 1,\ \ x_1 = 0$

(b) $x_0 = 1,\ \ x_1 = -1$

State clearly any results you use.

Problem 2

State necessary and sufficient conditions for a function $T(t, \sigma)$ to be the weighting pattern of a finite dimensional linear system.

Suppose you are given a *constant* matrix A of size $n \times n$ *and* a weighting pattern $T(t, \sigma)$ satisfying your conditions above.

Find a finite dimensional dynamical realization of the form

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B(t)\ u(t)\\ y(t) &= C(t)\ x(t)\end{aligned}$$

for $T(t, \sigma)$.
State clearly any *additional hypothesis necessary* for this purpose.

Problem 3 Consider the system (Σ)

$$\begin{aligned}\dot{x} &= Ax + bu\\ y &= cx + u\end{aligned}$$

with scalar input and scalar output.
Show that the system $(\tilde{\Sigma})$ given by

$$\begin{aligned}\dot{x} &= (A - bc)x + by\\ u &= -cx + y\end{aligned}$$

is *inverse* to the original system (Σ) in the sense that

$$\left[c\left(sI - A\right)^{-1} b + 1\right]\left[1 - c\left(sI - A + bc\right)^{-1} b\right] \equiv 1$$

Problem 4

Define McMillan degree of a transfer function.

A designer claims to have *built* a *controllable* and *observable* realization of the transfer function

$$R(s) = \frac{1}{s+4}\begin{bmatrix} 1 & 2 & 1\\ 2 & 4 & 0\end{bmatrix}$$

on a state space of dimension 3.
A second engineer claims to have built a *controllable* and *observable* realization of the transfer function

$$\tilde{R}(s) = \frac{1}{s+4\cdot 01}\begin{bmatrix} 1 & 2 & 1\\ 2 & 4 & 2\end{bmatrix}$$

on a state space of dimension 2.

Who is right? State clearly any results you use to support your answer.

Problem 5

Consider a transfer function $R(s)$ with the realization

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

with

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}; \qquad B = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}; \qquad C = [1 \;\; 1] \, .$$

(a) What Kronecker invariants would you associate with this system?

(b) Is there an *output feedback* yielding a *single nonzero* Kronecker invariant for the closed loop system?

(c) Is the given realization minimal? If not, produce one.

Problem 6

Consider the linear time-invariant system

$$\begin{aligned} \dot{x} &= Ax + bu \\ y &= cx \end{aligned}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & 0 & 0 \end{bmatrix}; \qquad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \qquad c = [2 \;\; 0 \;\; 0]$$

(i) Suppose you have access to full state. How would you design a controller such that all the eigenvalues of the system shift to -1?

(ii) Suppose you only have access to input and output. Suggest a controller structure and a design to achieve the goal of part (i).

Problem 7

State the *Laurent* series expansion of the transfer function of the linear system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

in terms of the coefficients A, B, C.
For the special case

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad C = (1 \;\; 0)$$

compute the first 10 terms in the Laurent series expansion.
State clearly any key results you use in this computation.