Electrical and Computer Engineering Department
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ENEE 660 – System Theory

Mid Term Exam I, 3:30-4:45 p.m.
Thursday, October 7, 2010

Closed book exam. Answer all questions.

Problem 1

Consider a linear map $A : \mathbb{R}^3 \to \mathbb{R}^3$ defined by the matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 0 \\ 2 & 0 & 3 \end{bmatrix}$$

Compute the matrix representation of this linear map in the basis $\{\upsilon_1, \upsilon_2, \upsilon_3\}$ defined by

$$\upsilon_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \upsilon_2 = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} \quad \upsilon_3 = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}.$$

Does this new matrix have any pure imaginary eigenvalues?

Problem 2

Give a clear and complete definition of the concept of adjoint of a linear map.
Consider

$$\mathcal{U} = \left\{ u : [0,2] \to \mathbb{R}^2 |\ u(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} \text{ is a continuous function} \right\}$$

Let $A : \mathcal{U} \to \mathbb{R}^3$ be the linear map defined as

$$Au = \begin{bmatrix} \int_0^2 u_1(r)dr & \\ \int_0^2 3u_1(r)dr & -\int_0^1 u_2(r)dr \\ -\int_0^{3/2} u_2(r)dr & \end{bmatrix}$$

Compute the adjoint of A. State clearly any hypotheses you need for this purpose.

Problem 3

Consider the linear time-varying system

$$\dot{x}(t) = e^{-tA} B\, e^{tA}\, x(t)$$

where A and B are constant square matrices. *Show* that a solution to the given system with initial condition $x(t_0) = x_0$ is given by

$$x(t) = e^{-tA}\, e^{(t-t_0)(A+B)}\, e^{t_0 A} x_0$$

Compute this in the special case,

$$\begin{aligned} A &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ B &= \begin{pmatrix} 3/2 & 0 \\ 0 & 3/2 \end{pmatrix} \\ t_0 = 1; \quad x_0 &= \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \end{aligned}$$

Problem 4

Consider the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

(a) Give a clear justification as to whether or not it is possible to drive the system from $x_1(0) = 1,\ x_2(0) = 2$, to $x_1(1) = x_2(1) = 0$.

(b) If your answer to part (a) is in the affirmative, determine a control that does the prescribed transfer.

Problem 5

Consider a *linear time-invariant* system of the form

$$\dot{x} = (A + \epsilon B)x$$

where A and B are known but ϵ is a small uncertain parameter. It is of interest to find the *sensitivity* of the solution at $t = 1$,

$$\left.\frac{\partial x(t)}{\partial \epsilon}\right|_{\substack{t=1 \\ \epsilon=0}},$$

for a given initial condition $x(0)$. Solve this problem in the special case,

$$\begin{aligned} A &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ B &= \begin{pmatrix} 3/2 & 0 \\ 0 & 3/2 \end{pmatrix} \\ \text{and } x(0) &= \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{aligned}$$