Electrical and Computer Engineering Department
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ENEE 660 – System Theory

Mid Term Exam II, 3:30-4:45 p.m.
Thursday, November 11, 2010

Closed book exam. Answer all questions.

Problem 1

(a) Consider a linear time-varying system

$$\dot{x}(t) = A(t)\, x(t)$$

with $A(t+T) = A(t)$ for a specific $T > 0$ and $\forall\, t \,\epsilon\, \mathbb{R}$. Under what conditions does this system admit a nontrivial periodic solution of period T?

(b) Show that the equation

$$\dddot{x}(t) + \ddot{x}(t) + \dot{x}(t) + x(t) = sin(2t)$$

has a unique periodic solution of period π. State clearly any results you use.

Problem 2

(a) Consider the linear time-varying system

$$\begin{aligned} \dot{x}(t) &= A(t)\, x\,(t) + B(t)\, u\,(t) \\ y(t) &= C(t)\, x\,(t) \end{aligned}$$

State clearly what the weighting pattern associated to this system is, and how it determines a relation between input and output.

(b) Under what conditions can a given function $T(t, \sigma)$ be the weighting pattern of a linear system of the form in (a) above, *but* with time-invariant coefficients?

(c) Consider the equation

$$\ddot{y} + 3\dot{y} + 2y = \dot{u} + 2u$$

determining a mapping between scalar signals $u(t)$ and $y(t)$. Construct this mapping in terms of a state space realization and associated weighting pattern.
Indicate whether your choice above has minimal state space dimension.

Problem 3

Consider the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

(a) When the control is turned off do all solutions of the above system go to 0 as $t \to \infty$? Does any solution go to 0 as $t \to \infty$?

(b) If your answer to *any* portion of (a) is **NO**, how would you proceed to find a *state feedback* that makes all solutions of the closed loop system go to 0 as $t \to \infty$?

(c) If you only have access to an output signal $y = x_1$ can you still use feedback to meet the objective of part (b)? Explain steps if your answer is **YES**.

Problem 4

You are given a linear time-invariant system of the form

$$\begin{aligned} \dot{x} &= Ax + bu \\ y &= cx \end{aligned}$$

with n states, 1 input and 1 output.
You are also told:

(i) Rank $\left(\int_0^1 e^{A\sigma} b \, b^T e^{A^T \sigma} \, d\sigma \right) = n - 1$

(ii) $[A, c]$ is observable.

Which of the following statements is true?

(a) There is a control of the form $u(t) = k(t)\ x(t) + v(t)$ such that you can drive the given system from 0 to any final state at time 1.

(b) For *any* such control as in part (a) you can reconstruct state history over time interval [0, 1] from output history over same time interval.

Your answers must have clearly stated justification.