ENEE 660 Homework Set #1

1. Let V and W be vector spaces, and let $A : V \to W$ be a linear map. Let $\mathrm{im}(A) = \{ w \in W : w = Av \text{ for some } v \in V \}$ (the image of A or also the range of A). Let $\ker(A) = \{ v \in V : Av = 0 \}$, (the kernel of A or the null-space of A).

~~(a)~~ Show that $\mathrm{im}(A)$ and $\ker(A)$ are vector subspaces of W and V respectively.

~~(b)~~

2. Let V be a vector space of $\dim(V) = n$. Suppose $S = \{ v_1, v_2, \cdots, v_n \}$ and $S' = \{ v_1', v_2', \cdots, v_n' \}$ are two different bases for V. Let

$$T = [t_{ij}]$$

denote a matrix (with n rows and n columns) such that

$$v_j' = \sum_{i=1}^{n} t_{ij} v_i \qquad j = 1, 2, \ldots, n.$$

(a) Show that T is unique.
(b) Show that T is invertible.

3. Let U, V and W be three finite dimensional vector spaces.

Let $A : U \to V$ ~~to~~ and

$B : V \to W$

be two linear maps.

Suppose you are given bases $S_U \subset U$, $S_V \subset V$ and $S_W \subset W$. Further, with these bases A has matrix representation $\tilde{A}$ and B has matrix representation $\tilde{B}$. Consider the map $C : U \to W$, defined by

$$C(u) = B(A(u))$$

(a) Show that C is linear

(b) Determine ~~the~~ the matrix representation of C in the given bases.

4. Consider the matrix

$$\tilde{A} = \begin{pmatrix} 0 & 0 & \cdots & 0 & -a_n \\ 1 & 0 & \cdots & 0 & -a_{n-1} \\ 0 & 1 & 0 & \cdots & -a_{n-2} \\ \vdots & & \ddots & & \\ 0 & & & 1 & -a_1 \end{pmatrix}$$

show that its characteristic polynomial is

$$\chi_{\tilde{A}}(\lambda) = \lambda^n + a_1 \lambda^{n-1} + \cdots + a_{n-1}\lambda + a_n$$