ENEE 660 Fall 2010 Homework 3 (due in class Thursday, Sept 23)

1. Suppose a scalar function $x(t)$ satisfies the nth order scalar differential equation,

$$x^{(n)}(t) + p_{n-1}(t)\,x^{(n-1)}(t) + p_{n-2}(t)\,x^{(n-2)}(t) + \cdots + p_0(t)\,x(t) = 0$$

where $x^{(k)}(t)$ denotes the k^{th} derivative of $x(t)$. Show that there is a choice of n dimensional state vector y with components

$$y_i = x^{(i-1)} \qquad i = 1, 2, \ldots, n$$

such that

$$\dot{y} = Ay$$

what is A?

Suppose

$$p_{n-k}(t) = q_{n-k}\, t^{-k}, \qquad k = 1, 2, \cdots, n$$

where q_i are all constant. Find a choice of state vector y such that

$$\dot{y} = \frac{1}{t} A y \qquad y \in \mathbb{R}^n$$

and A a constant $n \times n$ matrix. What is A?

2. (i) Verify that the transition matrix for the system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2\delta \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

with $0 \le \delta < 1$,

takes the form

$$\Phi(t,0) = \begin{bmatrix} e^{-\delta t}\left(\cos(\omega t) + \frac{\delta}{\omega}\sin(\omega t)\right) & \frac{1}{\omega} e^{-\delta t}\sin(\omega t) \\ -\frac{1}{\omega} e^{-\delta t}\sin(\omega t) & e^{-\delta t}\left(\cos(\omega t) - \frac{\delta}{\omega}\sin(\omega t)\right) \end{bmatrix}$$

where $\omega = \sqrt{1-\delta^2}$

(ii) Find a closed form expression for the transition matrix of

$$A(t) = \begin{pmatrix} a(t) & b(t) \\ -b(t) & a(t) \end{pmatrix}$$

3. Suppose $A(t) = [a_{ij}(t)]$ satisfies

$$a_{ij}(t) \geq 0 \quad \text{for all } i \neq j$$

and all $t > t_0$.

Show that every element of the transition matrix $\Phi(t, t_0)$ is ≥ 0 for all $t \geq t_0$.

Also demonstrate that in this case,

for the two systems

$$\dot{x}(t) = A(t)\, x(t) \qquad x(t_0) = x_0$$
$$\dot{y}(t) = A(t)\, y(t) \qquad y(t_0) = y_0$$

$$(x_0)_i \geq (y_0)_i \quad i = 1, 2, \ldots, n \quad \Rightarrow \quad x_i(t) \geq y_i(t)$$
$$i = 1, 2, \ldots, n.$$

4. Suppose $A(t)$ is a <u>skew symmetric</u> matrix for each t. In that case, show that the corresponding transition matrix $\Phi(t, t_0)$ has the property of <u>preserving inner product</u> as defined in Homework 2.