ENEE 660 Fall 2010 Homework #5

to be returned TUESDAY Oct 12.

1. Let A and B be constant $n \times n$ matrices. Let,

$$P(\varepsilon) = \frac{d}{d\varepsilon} e^{A + \varepsilon B}$$

Find the first three (3) terms in the power series for $P(\varepsilon)$.
(Hint: Consider problem 2 below)

2. Convert the differential equation

$$\dot{x}(t) = A\,x(t) + B(t)\,x(t)$$
$$x(0) = x_0$$

into the integral equation

$$x(t) = e^{tA} x_0 + \int_0^t e^{(t-\sigma)A} B(\sigma)\,x(\sigma)\,d\sigma$$

3. Consider a harmonic oscillator with a driving input $u(t)$ satisfying the equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

(a) Compute the reachability Gramian $W(0,T)$ for this system.

(b) Suppose we want to drive the system from the state $\begin{pmatrix}1\\0\end{pmatrix}$ to the state $\begin{pmatrix}0\\0\end{pmatrix}$ in 2π units of time.

Does there exist a control u which can accomplish this? If so find one.

(c) Suppose u is required to be a piecewise constant function of the form;

$$u(t) = \begin{cases} u_1, & 0 \le t < \frac{2\pi}{3} \\ u_2, & \frac{2\pi}{3} \le t < \frac{4\pi}{3} \\ u_3, & \frac{4\pi}{3} \le t \le 2\pi \end{cases}$$

Do there exist constants u_1, u_2, u_3 such that we can make the transfer from $\begin{bmatrix}1\\0\end{bmatrix}$ at $t=0$ to $\begin{bmatrix}0\\0\end{bmatrix}$ at time $t = 2\pi$?