ENEE 660 Fall 2010 Homework 7 due in class Tuesday, November 2

1. Let $M(t_0, t_1)$ denote the observability Gramian

$$M(t_0,t_1) = \int_{t_0}^{t_1} \Phi^T(\sigma, t_0)\, C^T(\sigma)\, C(\sigma)\, \Phi(\sigma, t_0)\, d\sigma$$

where Φ denotes the transition matrix for the linear system

$$\dot{x}(t) = A(t)x(t) \quad ; \quad x(t_0) = x_0 .$$

$$y(t) = C(t)\, x(t)$$

Show that

(a)
$$\frac{d}{dt} M(t, t_1) = -A^T(t) M(t,t_1) - M(t,t_1) A(t) - C^T(t) C(t)$$

$$M(t_1, t_1) = 0 .$$

(b)
$$M(t_0, t_1) = M(t_0, t) + \Phi^T(t, t_0) M(t,t_1) \Phi(t,t_0)$$

2. Show controllability of $[A, B]$ is equivalent to

$$\text{rank} \underbrace{[sI - A \quad B]}_{n \times (n+m) \text{ rectangular matrix}} = n$$

3 Consider the oscillator

$$\ddot{x}(t) + x(t) = 0 .$$

Can $x(0)$ and $\dot{x}(0)$ be determined from knowledge of the value of $x(t)$ at $t = \pi, 2\pi, 3\pi, \ldots$ etc. ?

4. Consider a linear-time invariant system

$$\dot{x}(t) = A\, x(t)$$
$$y(t) = C\, x(t)$$

Suppose $y(t)$ is measured only at particular ~~instance~~ time instants, $t = 0, T, 2T, 3T, \ldots$, for a suitable sampling time T. Determine conditions such that $x(0)$ can be determined uniquely from sufficiently many such samples.

[Hint : Consider a result analogous to that stated in problem 2.]