Notes related to discussions
in my office with students
from ENEE 660 on
Tuesday, November 9, 2010.

P. S. Krishnaprasad
10/10/2010

In theorem 2 page 100 of Brockett FDLS we use the property

$$(N W_T N^T)^{-1} \text{ exists.}$$

Why?

Proof: For notation see text of Brockett (and my classnotes / your lecture notes).

Let $\bar{W}_T \triangleq N W_T N^T \qquad n_0 \times n_0$

$\bar{M}_T \triangleq R^T M_T R \qquad n_0 \times n_0$

Recall $W_T = P S_1 P^T ; \quad M_T = Q^T S_2 Q$

$$\bar{M}_T \bar{W}_T = R^T M_T R N W_T N^T$$

$$= R^T Q^T S_2 Q R N P S_1 P^T N^T$$

$$= R^T Q^T S_2 Q Q^{-1} S_2 Q P S_1 P^{-1} P S_1 P^T N^T$$

(since $RN = Q^{-1} S_2 Q P S_1 P^{-1}$)

$$= R^T Q^T S_2^2 Q P S_1^2 P^T N^T \qquad (\because S_i^2 = S_i)$$

$$= R^T Q^T S_2 Q P S_1 P^T N^T$$

$$= R^T Q^T Q Q^{-1} S_2 Q P S_1 P^{-1} P P^T N^T \qquad (\because I = QQ^{-1} = P^{-1}P)$$

$$= R^T Q^T Q R N P P^T N^T$$

$$= (QR)^T (QR) \quad (NP)(NP)^T$$

But $(QR)^T$ is of rank n_0 (full rank), and $(QR)^T(QR)$ is the associated $n_0 \times n_0$ Gramian. So $(QR)^T(QR)$ is positive definite.

Similarly $(NR)(NP)^T$ is the $n_0 \times n_0$ Gramian of the full rank $n_0 \times n$ matrix (NP). So $(NP)(NP)^T$ is positive definite.

Hence the product $\bar{M}_T \bar{W}_T$ is <u>nonsingular</u>. Hence each of $\bar{M}_T$ and $\bar{W}_T$ are nonsingular.

Hence we have shown that $(N W_T N^T)^{-1}$ exists $\boxtimes$