$$\{x \in \mathbb{R}^n \mid V(x) \le c\} = \Omega_c \subset B_r .$$

Thus Ω_c is bounded, (and closed). By LaSalle's invariance principle $x(t)$, a trajectory beginning in Ω_c, $\to M$ = largest invariant set $\subset E = \{x \mid \dot V = 0\}$, as $t \to \infty$.

By (iii) $M = \{0\}$.

Thus 0 is asymptotically ~~stability~~ stable and globally attractive. $\boxtimes$

Chetaev ≠ Cetaev = Chetayev

The idea of using "energy-like" functions can be used also to prove instability theorems. A fundamental result is due to Nikolai Guryevich Chetaev (1902-1959), a Soviet mechanician, who held the chair of Theoretical Mechanics at Moscow State University.

Theorem (Cetaev)

Let $x = 0$ be an equilibrium point of $\dot x = f(x)$. Let $V : D \to \mathbb{R}$ be a C^1 function defined on an open, connected subset D of $\mathbb{R}^n$ which contains 0, and such that:

(i) $V(0) = 0$ and for every $\varepsilon > 0$, there exists $x_0 \in B_\varepsilon(0)$ such that $V(x_0) > 0$.

(ii) Let $U = \{x \in B_r(0) \mid V(x) > 0\}$ for $B_r \subset D$ and $\dot V(x) \geq \phi(\|x\|) > 0$ on U