ENEE 661 Spring 2011 Homework 1
due 02/10/2011

1 Show that the collection of all $4 \times 4$ matrices of the form

$$\begin{bmatrix} 1 & a_{12} & a_{13} & 0 \\ 0 & 1 & a_{23} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$$

with $a_{44} \neq 0$ is matrix (Lie) group.
Determine a basis for its Lie algebra.
In your choice of basis determine the structure constants.

2 Show that the smallest Lie algebra of matrices which contains $A_1, A_2$ with

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

is four-dimensional.

3 Let $t \mapsto \Phi(t)$ be a smooth curve in $Sl(n, \mathbb{R})$.
Show that we can write

$$\dot{\Phi}(t) = \Phi(t)\, \xi(t)$$

where $\xi(t) \in sl(n, \mathbb{R})$ i.e. the space of $n \times n$, matrices of trace $= 0$.

4 Let $t \mapsto \Phi(t)$ be a smooth curve in $SE(n, \mathbb{R})$ the Euclidean group of matrices of the form

$$\left[\begin{array}{c|c} A & b \\ \hline 0\,0\,0 & 1 \end{array}\right] \qquad \text{where } A \in SO(n)$$

$b \in \mathbb{R}^n$ and we have a row of $n$ zeros beneath the matrix $A$. Show that the associated Lie algebra is made up of matrices of the form

$$\left[\begin{array}{c|c} \Omega & \eta \\ \hline 0.0.0 & 0 \end{array}\right]$$

where $\Omega = -\Omega^T$ and $\eta \in \mathbb{R}^n$.

For $n = 3$ determine all structure constants in a suitable/natural basis for this Lie algebra, denoted as $se(3, \mathbb{R})$